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A Qualitative Approach to Direct Inference Principle

Mohamed Yasser Khayata
Department of Computer Science
Angers University, France

Jihad Mohamad Jaam
Department of Computer Science
Qatar University, Qatar

طريقة نوعية مقترحة لمبدأ الاستنتاج المباشر

محمد ياسر خياطة
كلية العلوم، قسم علوم الحاسب
جامعة أنجة - فرنسا

جهاد محمد الجعم
كلية العلوم، قسم علوم الحاسب
جامعة قطر - قطر

في هذا البحث، تمت دراسة ملائمة النموذج الرمزي للاستنتاج المنطقي الثلاثي والاستنتاج المباشر لتمثيل وإدارة المعرفة التي تعتمد على المعلومات الإحصائية المقيمة نوعياً، لأجل ذلك نقدم طريقة معتمدة بشكل رئيسي على المنطق الرمزي المحدود حيث أن مقياس التدرج في هذا المنطق ممثل بدرجات حقائق منطقية مختلفة، كما نقدم تمثيل حدسي وبديهي لبعض المفاهيم والقواعد الأساسية لهذا النموذج، ثم نقوم باستخدام بعض خصائصه الرئيسية لحل بعض المسائل الكلاسيكية في اللغة الطبيعية، ونبين كذلك أن الاستنتاجات التي حصلنا عليها هي في توافق جيد مع تلك الصادرة عن الاستنتاجات المنطقية.

Key Words: *Linguistic Quantifiers, Statistical Probability, Syllogism, Direct Inference, Certainty.*

ABSTRACT

In this paper,we study the suitability of a symbolic model of syllogistic reasoning and direct inference principle with the representation and the management of knowledge based on statistical information and evaluated in a qualitative way.Our ap-proach is founded mainly on a symbolic finite M-valued logic,in which the graduation scale of M symbolic quantifiers is translated in terms of truth degrees.We propose an intuitive presentation of some basic notions of this model.We use its main properties to solve some classical problems expressed in the natural language.We show that the obtained deductions are in a good accordance with those resulting from common sense reasoning.

1. Introduction

In our previous papers [8,9], we have studied the management of knowledge containing statisti-cal information and expressed in the natural lan-guage. The representation of statistical informa-tion is made with the aid of quantified statements which refer to adverbial expressions (i.e., linguistic quantifiers here)like "all", "almost all","most", "few", etc. It is obvious that a representation model will be interesting if an inference process, like the syllogistic reasoning [24], allows to deduce new quantified assertions. For example, knowing that "most students are young" and "almost all young students are unmarried", we wish to deduce that "most students are young or unmarried". It is also interesting that another inference process, like the direct inference principle [2,11,17], allows to deduce,from a set of quantified assertions, new information about the properties of a particular individual of the domain. For example,we wish to deduce a new piece of information about the bird "Tweety"knowing that "most birds fly". This can be the statement "it is very probable that Tweety flies".In other words,the certainty degree "very probable"will be associated with the state-ment "Tweety flies".Many approaches of these problems are generally based on probability theory [1, 2, 3, 4, 5, 11, 15, 17] or fuzzy set theory [5, 6, 21 , 22 , 24]. In [8, 9], we have proposed a formal model of syllogistic reasoning based on the substrate of the M-valued predicate logic intro-duced by Pacholczyk in [16].The resulting model allows us to reason with particular individuals by using knowledge based on quantified assertions via the direct inference principle.

The main objective of this paper is to propose an informal presentation of the basic concepts of the previous model,and to study the suitability of this theory with the representation and the management of linguistic knowedge encoded in a qualitative way. In Section 2, we briefly present the model given in our papers [8, 9] to reresent and manage quantified assertions with the aid of a syllogistic inference process.In Section 3, we recall the symbolic subjective probabilities proposed by Pacholczyk in [16]. Section 4 deals with a symbolic formalization of direct inference principle and a choice strategy of the appropriate reference class allowing us to reason with particular individuals of the basic domain. Finally, in Section 5, we make a comparison with other related approaches, mainly the probabilistic works of Bacchus et al. [2,3].

2. Presentation of The Symbolic Proportion

Psychologists consider that the symbolic graduation cannot be reasonably apprehended by human above ten degrees [12, 13, 18]. So, our graduation scales contains only seven adverbial expressions (M = 7). A first scale $\mathfrak{L}_7$ of degrees of *truth* enablesus to expresss the *graduation of vagueness*: "*John is very tall* " is equivalent to say that John satisfies the predicate "*tall* " to the degree "*very*". A second scale $\mathfrak{Q}_7$ of degrees of statistical probability allows us to express the graduation of proportion:given the basic space Ω,"Q_α Ω's are A's"means that an absolute proportion Q_α of individuals of Ω are in A with respect to the uniform probability distribution on Ω. A third scale U_7 of degrees of *certainty* is used to express the *graduation of certainty*: "*it is very probable that Tweety flies*" means that "*very-probable*" is the certainty degree of the assertion "*Tweety flies*". The graduations scales are as follows (with M = 7):

- $\mathfrak{L}_7 = \{T_\alpha \mid \alpha = 1,7\}$ = {*Not-at-all-true, Very-little- true, Little-true, Moderately-true, Very-true, Almost-true, Totally-true*}[1] ,
- $\mathfrak{Q}_7 = \{ Q_\alpha \mid = 1,7 \}$ = {*None, Very-few, Few, About-half, Most, Almost-all, All*}, and
- $U_7 = \{u_\alpha \mid \alpha = 1,7\}$ = {*Not-at-all-probable, Very-little-probable, Little-probable, Moderately-probable, Very-probable, Almost-certain, Certain*}.

As mentioned previously, the semantic model of statistical probabilities used here is built on the sub-strate of the M-valued predicate logic proposed by Pacholczyk in [16]. Let us briefly recall the ba-sic notions of interpretation and satisfaction of this M-valued logic. Let $M \geq 2$ be an odd integer. Let $\mathfrak{M}$ be the integer interval [1, M] totally ordered by the relation $\leq$, and let *n* be the mapping defined by $n(\alpha) = M + 1 - \alpha$. Then, $\{\mathfrak{M}, \vee, \wedge, n\}$ is a De Morgan lattice with: $\alpha \vee \beta = \max(a, \beta)$ and $\alpha \wedge \beta = \min(\alpha, \beta)$. Let $\mathfrak{L}_M = \{\tau_\beta \alpha \in \mathfrak{M}\}$ be a set of *M* elements totally ordered by the relation $\leq$: $\tau\alpha \leq \tau\beta \Leftrightarrow \alpha \leq \beta$. Thus $\{\mathfrak{L}_M, \leq\}$ is a chain in which the least element is τ_1 and the greatest element is τ_M. We define in $\mathfrak{L}_M$ the fol-lowing operators: $\tau_\alpha \vee \tau_\beta = \tau_{\max(\alpha,\beta)}$ $\tau_\alpha \wedge \tau_\beta = \tau_{\min(\alpha,\beta)}$ and $\sim \tau_\alpha = \tau_{n(\alpha)}$. We can interpret $\mathfrak{L}_M$ as a set of linguis-tic truth degrees dealing with vague predicates. By choosing $M = 7$, we can introduce the previous set $\mathfrak{L}_7$. We call an *interpretation structure* $\mathfrak{A}$ of the M-valued predicate language $\mathfrak{L}$, apair $< \mathfrak{D}, I >$, where $\mathfrak{D}$ is the *domain* of $\mathfrak{A}$ and *I* the interpretation function. We denote by R_n the multiset[2] associated with the predicate P_n. We call a valuation of variables, a sequence denoted by $s = <s_0, \ldots, s_{i-1}, s_i, s_{i+1}, \ldots,>$ with $s_i \in \mathfrak{D}$. The valuation s(i/a) is defined by the following: $s(i/a) = <s_0, \ldots, s_{i-1}, a, s_{i+1}, \ldots,>$.

Definition 1: The relation of partial satisfaction **s satisfies a formula Φ to a degree τ_α in-$\mathfrak{A}$** or **s τ_α-satisfies Φ in-$\mathfrak{A}$** , denoted by $\mathfrak{A} \models^s_\alpha \Phi$, is defined as follows:

- $\mathfrak{A} \models^s_\alpha P_n(z_{i1}, \ldots, z_{ik}) \Leftrightarrow <s_{i1}, \ldots, s_{ik}> \in \alpha R_n$.
- $\mathfrak{A} \models^s_\alpha \neg \phi \Leftrightarrow \mathfrak{A} \models^s_\beta \phi$ with $\tau_\alpha = {}^\sim \tau_\beta$,
- $\mathfrak{A} \models^s_\alpha \phi \cap \Psi \Leftrightarrow \{\mathfrak{A} \models^s_\beta \phi$ and $u = {}^s_\gamma \Psi$ with $\tau_\alpha = \tau_\beta \vee \tau_\gamma\}$,
- $\mathfrak{A} \models^s_\alpha \phi \cup \Psi \Leftrightarrow \{{}^s_\beta \phi$ and $u \models^s_\gamma \Psi$ with $\tau_\alpha = \tau_\beta \vee \tau_\gamma\}$,
- $\mathfrak{A} \models^s_\alpha \exists z_n \Psi \Leftrightarrow \tau_\alpha = \mathrm{Max}\{\tau_\gamma \mid \mathfrak{A} \models^s_\gamma \Psi$ with $T_\alpha = \tau_\beta \rightarrow \tau_\gamma\}$,
- $\mathfrak{A} \models^s_\alpha \forall z_n \Psi \Leftrightarrow \tau_\alpha = \mathrm{Max}\{\tau_\gamma \mid \mathfrak{A} \models^{s(n/a)}_\gamma \Psi, a \in D\}$,

Φ is said to be t_α – true-in-$\mathfrak{A}$, if the valuation s τ_α-satisfies Φ in-$\mathfrak{A}$

In order to facilitate the presentation of the basic notions of our formal model, all its definitions as well as its properties will receive more informal but more explicit linguistic translations.Note that the formal model can be found in our papers [8,9].

Let us now present the apparatus allowing us to handle with the previous statistical information. Linguistically speaking, our representation process refers to quantified assertions which receive the fol-lowing form "Q_μ A's are B's", where A and B denote sets, and Q_μ is a linguistic quantifier interpreted as a relative (or conditional) proportion evaluated in a qualitative way (with respect to the uniform probability

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[1] *Note that "not-at-all-true" and "totally-true" correspond respectively to "false" and "true".*

[2] *In the multiset theory [1], $x \in_\alpha A$, the membership de-gree to which x belongs to A, corresponds to $\mu A(x) = a$ in the fuzzy set theory[23].*

distribution on the universe of discourse Ω). More precisely, among the elements of Ω which belong to A, a proportion Q_μ of these elements belong to B. In other words, this definition can be viewed as a symbolic generalization of classical conditional probability: given the basic space Ω, Q_μ is defined by the symbolic division of Q_λ the symbolic absolute proportion of the intersection $A \cap B$ by Q_α the absolute proportion of A. We have generalized the classical definition of conditional statistical probability[3] in a symbolic context, by using a new predicate[4] with a "symbolic probabilistic division" operator, denoted by **C**, or equivalently a "symbolic probabilistic multiplication" operator, denoted by **I**. These two operators have been defined by Pacholczyk in [16]. The operator **I** verifies the classical properties of the probabilistic multiplication. The operator **C** is deduced from I by a unique way as follows: $Q_\mu \in C(Q_\alpha, Q_\lambda) \Leftrightarrow Q_\lambda = \mathbf{I}(Q_\alpha, Q_\mu)$. Among the different tables of the operator **C** (resp. **I**) we have chosen Table 1 (resp. Table 2) presented in the Annex section of this paper.

Definition 2 Given a basic space Ω:

- Any quantified assertion "Q_α Ω's are A's" means that an ***absolute proportion*** *Q_α of individuals of Ω are in A with respect to the uniform probability distribution on Ω.*

- Any quantified assertion "Q_μ A's are B's" means that among the elements of the basic space Ω which belong to A, a ***relative proportion*** *$Q\mu$ of these elements belong to B, and this with respect to the uniform probability distribution on Ω. It is defined as follows:*

if {Q_α Ω's are A's and Q_λ Ω's are $(A \cap B)$'s},then "Q_μ A's are B's" with $Q\mu \in C(Q_\alpha, Q_\lambda)$.

This proportion concept has to satisfy a set of axioms justified at a metalogical level (their formal presentation can be found in [8,9]).First of all, if A and $A \cap B$ represent the same significant absolute proportion,that means "almost all A's are B's" (axiom P1). Secondly (axiom P2), knowing that "almost-all A's are B's", then $A \cap B$ has the absolute proportion of A. Thirdly, A and A have symmetrical absolute proportions (axiom P3). Finally, the absolute proportion of $A \cup B$ results from the ones of A, B and $A \cap B$ (axiom P4). So, we obtain the following axioms[5]:

Axiom 1: $A \cap B \neq A$, "Q_α Ω's are A's" and "Q_α Ω's are $(A \cap B)$'s" and $Q_\alpha \in [Q_3, Q_7] \Rightarrow$ "Almost-All A's are B's". (Axiom 1 defines "Almost-all").

Axiom 2: "Q_α Ω's are A's", $Q_\alpha [Q_2, Q_7]$ and "Almost-All A's are B's" $\Rightarrow$ "Q_α Ω's are $(A \cap B)$'s". (Axiom 2 defines also "Almost-all").

Axiom 3: "Q_α Ω's are A's" $\Leftrightarrow$"$Q_{n(\alpha)}$ Ω's are A's with $n(\alpha) = M + 1 - \alpha$". (Axiom 3 defines the dual quantifier).

Axiom 4: "Q_α Ω's are A's", "Q_β Ω's are B's", $A \cup B \neq \Omega$ and $A \cap B = \phi \Rightarrow$ "Q_r Ω's are $(A \cup B)$'s" with $Q_r \in S(Q_\alpha, Q_\beta)$ where S is a symbolic addition. (Axiom 4 defines the symbolic proportion of disjoint sets union).

[3]*It appears clearly that we obtain a symbolic generalisa-tion of the classical properties:*

$Prob(A) = |A| / |\Omega|$ and

$Prob(B|A) = Prob(A \cap B) / Prob(A) = |A \cap B| / |A|$.

[4]*In the formal model, an M-valued predicate denoted by Prop stands for the function prob.*

[5]*Axiom 3 and Axiom 4 can be viewed as generalizations of classical results in probability theory.*

The "*symbolic sum*" denoted by S can be defined by Table 3 (see Annex)and the "*symbolic difference*" denoted by D can be deduced from S (see Table 4 in Annex).

Let A and B be subsets of **Ω**. We present some propositions which can be viewed as symbolic generalization propositions of classical statistical probabilities. The proofs of these properties can be found in our paper [9].

Proposition 1 (Properties)

– If "Q_α Ω's are A's" and A B, then "Q_β Ω's are B's" with $Q_\alpha \leq Q_\beta$.

– If "Q_α Ω's are A's", "Q_λ Ω's are $(A \cap B)$'s" and $A \neq \Omega$, then "Q_λ Ω's are $_\gamma$A\B's" with Q_γ Œ $D(Q_\alpha,Q_\lambda)$.

At this point,we can present our reasoning process on quantifiers,called by Zadeh [24] syllogistic reasoning that can be defined as follows:

Definition 3: a syllogism is an inference rule that consists of deducing a new quantified statement from quantified statements.

Let $\Re$ be the set of available quantified asser-tions, then we can deduce from $\Re$ by using syllogisms a set $\Re^*$ containing $\Re$ and new quantified assertions.In our approach we have the following syllogisms:

Proposition 2 (Syllogisms)

– **Relative Duality:** If $\Re$ contains "$Q_{\mu1}$ A's are B's" then $\Re$ contains "$Q_{\mu2}$ A's are B's" with $Q_{\mu2} = Q_{n(\mu1)}$ if $Q_{\mu1} \neq Q_{n(\mu1)}$ and $Q_{\mu2} \in [Q_{n(\mu1)}, Q_{n(\mu1)+1}]$ otherwise.

– **Mixed Transitivity:** If $\Re$ contains "$Q_{\mu1}$ A's are B's"and "All B's are C's"then $\Re$ contains "$Q_{\mu2}$ A's are C's"with $Q_{\mu1} \leq Q_{\mu2}$.

– **Intersection /Product Syllogism:** If $\Re$ contains "$Q_{\mu1}$ A's are B's"and "$Q_{\mu1}$ $(A \cap B)$'s are C's" then $\Re$ contains "Q_μ A's are $(B \cap C)$'s", with $Q_\mu = I(Q_{\mu1}, Q_{\mu2})$.

– **Intersection/Quotient Syllogism:** If "$Q_{\mu1}$ A's are B's", "$Q_{\mu2}$ A's are C's"and "$Q_{\mu3}$ $(A \cap B)$'s are C's" then Qμ $(A \cap C)$'s are B's", with $Q\mu \in C(Q_{\mu2}, I(Q_{\mu1}, Q_{\mu3}))$.

– **Contraction:** If $\Re$ contains "Almost-all A's are B's"and "Almost-all $(A \cap B)$'s are C's" then $\Re$ contains "Almost-all A's are C's".

– **Cumulativity:** If $\Re$ contains "Almost-all A's are B's"and "Almost-all A's are C's"then $\Re$ contains "Qμ $(A \cap B)$'s are C's",with $Q\mu \in$ [Most, Almost-all].

– **Union Left:** If $\Re$ contains "Almost-all A's are C's"and "Almost-all B's are C's"then $\Re$ contains "Qμ $(A \cup B)$'s are C's", with $Q_\mu \in$ [Most, Almost-all].

Example 1: Let us suppose R contains the following: "Almost all students are young"and "Almost all students are single". Then,by using the cumulativity syllogism,we can say that $\Re$ contains {"Qμ Young Students are Single"with $Q\mu \geq Q5$ } i.e."At least most young students are single".

3. Symbolic Presentation of Subjective Probabilities

We can now focus our attention on the second problem, which consists of dealing with particular individuals. More precisely, knowing that a particular individual "a" belongs to A (or "a is A"), we wish to deduce from "Qμ A's are B's" and available knowledge, a symbolic certainty degree to which the particular individual a belongs to B (or "a is B"). We introduce a certainty function **Cert** which is applied to boolean formulas.Thus, the statement "A(a) is u_α (or v_α-probable)" is translated into Cert (A(a)) = u_α.

Definition 4 Cert: (A(a)) = u_α is equivalent to say that "A(a) is v_α-probable" is totally true[6].

This certainty concept has to satisfy a number of postulates, each of them being justified at a metalogical level. First of all (Axioms C2 and C3),if a statement is true (resp.false),its certainty degree is certain (resp.impossible). If two statements are equivalent, they receive the same certainty degree (Axiom C1). The certainty degree of the negation is the symmetrical value in the graduation scale of the one of the affirmation (Axiom C4). Finally, if the intersection of two statements is false, then the cer-tainty associated with their union is the "symbolic sum" (Axiom C5)of their uncertainty (defined as T-conorm in [16]).So,The function Cert satisfies the following axiomatics:

C1: $A(a) \equiv B(a) \Rightarrow$ **Cert** $(A(a)) =$ **Cert** $(B(a))$

C2: A(a) is true $\Rightarrow$ **Cert** $(B(a)) = u_7$.

C3: A(a) is false $\Rightarrow$ **Cert** $(B(a)) = u_1$.

C4: **Cert** $(A(a)) = u_\alpha \Rightarrow$ **Cert** $(\neg A(a)) = u_{8-\alpha}$.

C5: {**Cert** $(A(a)) = u_\alpha$, **Cert**$(B(a)) = u_\beta$, **Cert** $(A(a) \cap B(a)) = u_1$} $\Rightarrow$ **Cert** $(A(a) \cup B(a))$ $= u_\gamma$ with $u_\gamma = S(u_\alpha, u_\beta)$.

4. Symbolic Direct Inference Principle

The quantified assertion "α % of individuals of the domain verify a property" can be interpreted as "the probability that a randomly selected domain individual satisfies the property is equal to α". This interpretation can be seen as a way of justifying the deduction of uncertain conclusions about particular individuals (i.e., subjective probabilities) from statistical knowledge (i.e., statistical probabilities) via the direct inference [2,19,11,17]. Indeed,the principle of direct inference is based on the idea that a particular individual in the domain is considered as a member randomly selected from a population, if no particular information distinguishes it from other members of this population. For example, if all we know about Tweety is that it is a bird,then Tweety can be viewed as a randomly selected member of the population of birds since we do not have any other information that distinguishes it from other birds. Thus, knowing that Tweety is a bird, the (subjective) probability that Tweety flies is equal to the (statistical) probability that a bird randomly selected from the set of birds flies, i.e. the proportion of flying birds among the birds.

We have proposed a symbolic generalization of the direct inference principle allowing us to infer a symbolic subjective probability degree from a symbolic statistical probability degree.

Definition 5: The available knowledge base can be formally represented in the basic domain by the couple KB = (W, $\mathfrak{R}$) where W is the conjunction of formulae representing the available knowledge about the particular individuals of the basic domain, and $\mathfrak{R}$ is the set of quantified assertions.

[6] *Strictly speaking, "A(a) is v_α-probable" should be totally true in the interpretation U. Moreover, in the M-valued predicate logic, the m-valued predicate Cert takes into account this uncertainty in the following way:*

$\mathfrak{A} = {}_\alpha$*Cert (a(a)), where A (a) denotes the boolena formula.*

– By using syllogisms, we can deduce from $\Re$ a set $\Re$ containing $\Re$ and new quantified assertions.

– W(a) will be the conjunction of formulas appearing in W mentioning a.

– W (a/z) is the formula obtained when textually substituting each occurrence of a in the formula W(a) by the free variable z.

– W(a/z) and B (a/z)denote respectively the sets associated with the formulas W (a/z)and B (a/z).

– Given a as an individual of the basic domain, a reference class of a given KB for a formula B (a) (in which we want to generate a certainty degree) is a subset of the basic domain to which belongs the individual a.

Intuitively, the substitution W(a/z) denotes a form related to the process of "random selection". The constant a is considered as a "random member" by replacing it in W(a)by the free variable z. This leads to suppose that the individual denoted by a is randomly chosen among the individuals sharing all its properties, i.e. the individuals satisfying W(a/z). Let us now present the basic notions leading to our basic definition of symbolic direct inference. Given a knowledge base KB, we suppose that a denotes an individual constant of the basic domain. Given a, we search its cer-tainty degree u_α-probable resulting from the available knowledge base KB = (W,R). It is defined in the following way:

Definition 6 (Direct Inference Principle)

Let us suppose that a denotes an individual con-stant of the domain, z a variable, and KB = (W, $\Re$) the available knowledge base.We say that Cert(B(a)) = u_α results from direct inference principle, if the quantified assertion {Q_α W(a/z)'s are B(a/z)'s} belongs to $\Re$*.

Example 2: Let us consider the following:

S1: Most native speakers of German are not born in America.

S2: All native speakers of Pennsylvanian Dutch are native speakers of German.

S3: Most native speakers of Pennsylvanian Dutch are born in Pennsylvania.

S4: All people which are born in Pennsylvania are born in America and Hermann is a native speaker of Pennsylvanian Dutch.

The knowledge base KB is made with KB = (W, $\Re$) where:

R={Q_5 German-speak(z)'s are America(z)'s,

Q_7 PDutch-speak(z)'s are German-speak(z)'s,

Q_5 PDutch-speak(z)'s are Pennsylv(z)'s,

Q_7 Pennsylv(z)'s are America(z)'s},W = P Dutch-speak (Hermann).

There is no syllogism allowing to deduce a quantified assertion from the quantified assertions S1 and S2. But by applying the syllogism of mixed transitivity, we have {Q_5 PDutch-speak(z)'s are Pennsylv(z)'s, Q_7 Pennsylv(z)'s are America(z)'s} $\Rightarrow$ $\Re$ contains {Q_μ PDutch-speak(z)'s are America(z)'s with $Q_\mu \geq Q_5$}. Then, the direct inference leads to deduce **Cert** (America (Hermann)) = Q_μ with $Q_\mu \geq$ Q5. In other words, "it is at least very-probable that Hermann is born in America".

Remark 1: It is important to point out that this direct inference principle is not always applicable especially when we don't have any meaningful information for the reference class W(a/z), i.e. we have {$Q_{[1,M]}$ W(a/z)'s are B(a/z)'s} (that is a case of total ignorance). Moreover,in some cases we may be confronted with the existence of conflictual reference classes.

4.1 Choice of a Reference Class

One is often confronted to the existence of conflictual reference classes.We can distinguish three conflict types:

- Conflict between less and more specific classes.
- Conflict between classes associated with less and more precise information.
- Conflict between incomparable classes.

To solve the first and the second conflict types, we are going to modify the basic definition of the direct inference by a symbolic formalization of the specificity rule of Reichenbach [19] and the strength rule of Kyburg [11]. For the third type, we are going to propose a combination function of symbolic degrees associated with incomparable reference classes.The specificity rule of Reichenbach [19] consists of choosing among reference classes, the smallest (specific) class for which we have meaningful information. We propose a symbolic formalization of the specificity rule allowing us to infer the certainty symbolic degree in B(a) from KB, by choosing information associated with the smallest reference class designed by W' (a/z).

Definition 7: (Specificity Rule)Let us suppose that: KB =(W,R). The specificity rule allows us to infer "Cert(B(a)) = u_α" if the three following conditions are satisfied:

1) We have {$Q_{[1,M]}$W (a/z)'s are B(a/z)'s} (i.e. from Q_1 to Q_M: total ignorance),

2) $\exists$ W' (a/z) such that $\Re$* contains {Q_7 W (a/z)'s are W' (a/z)'s} and {Q_α W'(a/z)'s are B(a/z)'s},

3) $\nexists$ W" (a/z)such that $\Re$ contains {Q_7 W" (a/z)'s are W' (a/z)'s} and {Q_β W"(a/z)'s are B(a/z)'s}.

Intuitively, the three conditions above express respectively the following:

(1) We don't have any meaningful information for the smallest reference class W(a/z). Otherwise, the corresponding definition will be used.

(2) The existence of a reference class W' (a/z)for which we possess a meaningful information.

(3) There is no smaller reference class W" (a/z) that W'(a/z) for which we possess a meaningful information.

Example 3: Let us suppose that we have the following knowledge base KB = (W, $\Re$):

$\Re$ = {Most elephants are gray: Q_5 Elephant(z)'s are Gry(z)'s, Few royal elephants are gray: Q_7 (Elephant(z) $\cap$ Royal(z))'s are Elephant(z)'s},

W = Clyde is an African royal elephant i.e.: Elephant (Clyde) $\cap$ Royal (Clyde) $\cap$ African (Clyde).

We have two reference classes having meaningful information, for Gray(Clude): Elephant(z) and Elephant(z) $\cap$ Royal(z). This last is the smallest class, because we have: (Q_7 (Elephant(z) $\cap$ Royal(z))'s are Elephant(z)'s) $\in$ $\Re$. Applying the specificity rule, we obtain: Cert(Gray(Clyde)) = u_3, i.e.,"it is little probable that Clyde is gray".

The strength rule of Kyburg [11] is used in the cases where information associated with reference classes are intervals.It considers that a class of reference is better than another one,if the associated information is more precise than the one associated with the other. In our symbolic context, we introduce the following definitions:

Definition 8: (Strength Rule) Let KB = (W, $\Re$). The strength rule,allows us to derive that: Cert(B(a)) = u_α with $u_\alpha \in [u_c, u_d]$, if the following conditions are satisfied:

- $\Re^*$ contains {$Q_{\alpha 1}$ W(a/z)'s are B(a/z)'s with Q ∈ 1 [Q_a,Q_b]},
- $\Re^*$ contains {$Q_{\alpha 2}$ W'(a/z)'s are B (a/z)'s with $Q_{\alpha 2} \in [Q_c, Q_d]$, and [$Q_c$,$Q_d$] [$Q_a$,$Q_b$]}.

Remark 2: The priority between the two rules is given by the strength rule. Therefore, the specificity rule can be applied when the strength rule condition is not verified.

The reference classes can be incomparable, i.e., none of these two rules can be used. Like in [3], the certainty degree results from a combination of certainty degrees associated with the incomparable reference classes.

A combination function Comb is an application of U^2_M into U_M possessing the following properties:

[Cb1]: $\forall_\alpha$, $\beta \in [2..M]$, Comb (u_α,u_β) = Comb (u_β,u_α), Commutativity.

[Cb2]: $\forall_\alpha$, $\beta \in [2..M]$, Comb $(u_\alpha,u_\beta) \in [u_{\min(\alpha,\beta)}, uu_{\max(\alpha,\beta)}$.

[Cb3]: $\forall_\alpha$, $\in [2..M]$, Comb $(u_\alpha,u_M) = u_M$, u_M is an absorbent element for any $\alpha \in [2..M]$.

[Cb4]: $\forall_\alpha$, $\in [1..M-1]$, Comb $(u_\alpha,u_1) = u_1$, u_1 is an
absorbent element for any $\alpha \in [1..M-1]$.

[Cb5]: $\forall_\alpha$, $\in [2..M-1]$, Comb $(u_\alpha,u_{n(\alpha)}) = u_4$, Conflict related to the ambiguity

[Cb6]: $\forall_\alpha$, $\beta\ \delta \in [1..M]$, Comb (Comb (u_α,u_β), u_δ) = Comb (u_α, Comb (u_β,u_δ)), Associativity.

[Cb7]: $\forall_\alpha$, $\beta\ \delta \in [2..M-1]$, Comb $(u_\alpha,u_\beta) = u_\delta \Rightarrow$ Comb $(u_\alpha,u_{\beta+1}) \in [u_\delta,u_{\delta+1}]$ Monotonicity.

We can choose the function Comb as follows:

$\forall_\alpha$, $\beta \in [2..m-1]$:

$$\textbf{Comb}\ (u_\alpha, u_\beta) = \begin{cases} u_{\lfloor(\alpha+\beta)/2\rfloor} \text{ if } \alpha+\beta \le M \\ u_{\lceil(\alpha+\beta)/2\rceil} \text{ if } \alpha+\beta > M \end{cases}$$

where $\lfloor r \rfloor$ (resp $\lceil r \rceil$) denotes the greatest integer lower than (resp. lowest integer greater than) or equal to r. We can choose the following table cor-responding to the function **Comb**.

If Q is associated with a numerical value (or a numerical interval [a,b]) then in Bacchus's approach, we can give the following syllogisms:

– 1 – Mixed Transitivity

Q A's are B's

1 B's are C's (1 is equivalent to 100 % or "All")

[Q,1]A's are C's (i.e.Q'A's are C's with $Q' \geq Q$).

Our approach gives the same result:

$Q_{\mu 1}$ A's are B's

All B's are C's

$Q_{\mu 2}$ A's are C's with $Q\mu 2 \geq Q\mu 1$.

-2 -Intersection/Product syllogism

Q1 A's are B's

Q2 $(A \cap B)$'s are C's

Q1*Q2 A's are $(B \cap C)$'s (where *stands for the multiplication operator).

Our approach gives a similar result since the operator **I** stands for an operator having in LM the properties of a multiplication operator [16].

$Q_{\mu 1}$ A's are B's

$Q_{\mu 2}$ $(A \cap B)$'s are C's

Q_{μ} A's are $(B \cap C)$'s, with $Q_{\mu} = I(Q_{\mu 1}, Q_{\mu 2})$.

It is clear that they correspond to the same syl-logisms since, for each syllogism, the resulting assertion is the same and the quantifier is obtained in the same way in the numerical and in the symbolic setting (that is using the same combination of the operators).

Moreover, the operators **C** (division), **I** (product), **S** (addition), **D** (difference) are the symbolic counterparts of the four classical operators (see Annex and the previous associated paragraph). The operators defined for the symbolic setting respect the properties of classical operator such as, depending on the considered operator, associativity, existence of a neutral element, commutativity, monotony, etc.

The behavior of our syllogistic reasoning when dealing with precise values is then depending on the symbolic operators used for the syllogism. The question is to verify that they are in accordance with the classical operators used in a numerical setting.

The problem is that it is not possible to prove that the symbolic operators are in total accordance with numerical operators. Indeed, there is no isomorphism between the numerical and the symbolic settings. So, it is not possible to give an interface between numerical and symbolic quantifiers allowing to compare the behaviors of the two different systems.

Finally, let us say that, on one hand, Bacchus' proposal is adapted when the given data are expressed with precise values but it would not be suitable when the information are symbolic. On the other hand, our work is useful when there is no precise values but when the information are only expressed in terms of symbolic values (which is suitable with the initial aim of our work).

5.1. Comparison with Bacchus Direct Inference

As noted before, Bacchus was interested in representing the quantifier "most" (denoting the majority). In our approach,this quantifier can be represented either by the quantifiers "most", "almost-all",or "all " that corresponds to "at least most ". Then,we obtain similar results like those found in Bacchus [2].In the set of examples used by Bac-chus, we does not give here the simplest ones but we focused only on the most important ones.

Let us now take the well-known Nixon Dia-mond: "Most Quakers are pacifist", "Most republicans are not pacifist", "Nixon is a republican Quaker". Bacchus does not decide whether Nixon is pacifist or not. In our framework, the same result is obtained (see example by using quantifiers Q5 instead of Q6 in order to use "Most" instead of "Almost-all") since we deduce both that "it is moderately probable that Nixon is pacifist "and "it is moderately probable that Nixon is not pacifist".

Another example is the following one:"Most native speakers of German are not born in America", "All native speakers of Pennsylvanian Dutch are native speakers of German", "Most native speakers of Pennsylvanian Dutch are born in Pennsylvania", "All people which are born in Pennsylvania are born in America" and "Hermann is a native speaker of Pennsylvanian Dutch". In [2], Bacchus deduced that the probability that Hermann is born in America is > 0.5,that is to say "it is probable that Hermann is American". In our framework, we find that "it is very probable that Hermann is born in America". The two results are in a good accordance. So, the results found in our framework are in accordance with those found in Bacchus'direct inference.

The notions of reference classes used in our paper are based on the same strategies used by Bacchus. Hence,we propose a definition of specificity rule and strength rule that is in accordance with the ones proposed by Bacchus. Moreover, for dealing with incomparable classes, we have introduced a combination function.

5.2. Comparison with Bacchus etal. s Direct Inference

The comparison with the work proposed recently by Bacchus et al. [3] needs a preliminary clarification. We have to notice that the notion of direct inference proposed in their paper differs from the one we use (and from the one proposed by Bacchus [2] in his previous work). Their aim is not the syllogistic reasoning but the default reasoning by defining an inference rule verifying postulates of rational inference relation. Their definition of direct inference is defined on the semantic of random worlds. In their work, they introduced a new operator to deal with quantifiers of the form "approximately x%". This allows to represent the quantifier "Almost-all", expressing a default rule, by "approximately 1". Their approach implies choice strategies of reference classes and a combination function of information associated with incomparable reference classes.Then,it is possible to compare their "numerical" results with the ones obtained with our approach.

As for the previous section,we only give here the comparison for one example (the one they use to explain the combination function). For "The Nixon Diamond" problem,our approach leads to "it is moderately probable that Nixon is pacifist". By putting, the quantifier associated with "Most Quakers are pacifists"and approximately equal to 1 and the quantifier associated with "Most republicans are pacifists" approximately equal to 0, they obtain the value 0.5 (when considering that the rate of exceptions is the same for each default).The deductions are in good accordance in both frame-works.

6. Conclusion

In this paper, we have informally presented a previous formal modelisation of syllogistic reasoning,direct inference principle and choice of the appropriate reference class. Then, we have studied its suitability with the representation and management of statistical knowledge encoded in a qualitative way. The different examples, generally refer to bench-mark problems presented in [20], illustrate the fact that our model works perfecty and leads to results in good accordance with the common sense reason-ing, i.e. results which are in good accordance with those classically obtained.So,as far as the comparison between the different frameworks is possible, we can say that the results we find are in accordance with the results found previousely by Bacchus [2] and Bacchus et al.[3]. This symbolic theory brings new tools to Artificial Intelligence and Linguistics for an explicit treatment of the uncertainty resulting from statistical information,and this is particularly true in instantiated reasoning. Since, in our approach, reasoning on particular individuals constitutes a non monotonic reasoning process, it will be interesting to verify that the properties associated with our process fullfil the basic postulates of a non monotonic relation,like the ones defining the system P [10].

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Annex: Tables of Operators

In the following, $Q_{a,b}$ stands for interval $[Q_a, Q_b]$.

Table 1 : Opertor C

C	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
Q_1	$Q_{1,7}$	0	0	0	0	0	0
Q_2	$\{Q_1\}$	$Q_{2,7}$	0	0	0	0	0
Q_3	$\{Q_1\}$	$Q_{2,5}$	$Q_{6,7}$	0	0	0	0
Q_2	$\{Q_1\}$	$Q_{2,4}$	$\{Q_5\}$	$Q_{6,7}$	0	0	0
Q_5	$\{Q_1\}$	$Q_{2,3}$	$\{Q_4\}$	$\{Q_5\}$	$Q_{6,7}$	0	0
Q_6	$\{Q_1\}$	$\{Q_2\}$	$\{Q_3\}$	$\{Q_4\}$	$\{Q_5\}$	$Q_{6,7}$	0
Q7	$\{Q_1\}$	$\{Q_2\}$	$\{Q_3\}$	$\{Q_4\}$	$\{Q_5\}$	$\{Q_6\}$	$\{Q_7\}$

Table 2 : Operator I

I	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
Q_1	Q_1	Q_1	Q_1	Q_1	Q_1	Q_1	Q_1
Q_2	Q_1	Q_2	Q_2	Q_2	Q_2	Q_2	Q_2
Q_3	Q_1	Q_2	Q_2	Q_2	Q_2	Q_3	Q_3
Q_4	Q_1	Q_2	Q_2	Q_2	Q_3	Q_4	Q_4
Q_5	Q_1	Q_2	Q_2	Q_3	Q_4	Q_5	Q_5
Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_6
Q_7	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7

Table 3 : Opertor S

S	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_1	$\{Q_1\}$	$\{Q_2\}$	$\{Q_3\}$	$\{Q_4\}$	$\{Q_5\}$	$\{Q_6\}$
Q_2	$\{Q_2\}$	$Q_{2,3}$	$Q_{3,4}$	$Q_{4,5}$	$Q_{5,6}$	$\{Q_6\}$
Q_3	$\{Q_3\}$	$Q_{3,4}$	$Q_{4,5}$	$Q_{5,6}$	$\{Q_6\}$	
Q_4	$\{Q_4\}$	$Q_{4,5}$	$Q_{5,6}$	$\{Q_6\}$		
Q_5	$\{Q_5\}$	$Q_{5,6}$	$\{Q_6\}$			
Q_6	$\{Q_6\}$	$\{Q_6\}$				

Table 4 : Opertor D

D	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_1	$\{Q_1\}$					
Q_2	$\{Q_2\}$	$\{Q_2\}$				
Q_3	$\{Q_3\}$	$Q_{2,3}$	$\{Q_2\}$			
Q_4	$\{Q_4\}$	$Q_{3,4}$	$Q_{2,3}$	$\{Q_2\}$		
Q_5	$\{Q_5\}$	$Q_{4,5}$	$Q_{3,4}$	$Q_{2,3}$	$\{Q_2\}$	
Q_6	$\{Q_6\}$	$Q_{5,6}$	$Q_{4,5}$	$Q_{3,4}$	$Q_{2,3}$	$\{Q_2\}$