Qatar Univ. Sci. J. (1995), 15 (2) : 267 - 277

# THRESHOLD MODELLING OF SHIP'S DYNAMICS

By

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# النمذجـــة العتبيـــة لديناميكيـــة السـفينة

باســـل يونـس ذنــــون

تتضمن هـذه المقالة تطبيق لنظام يُدعى بنظـام العتبة في مجال الهندسة البحريـة . وتُناقش خصاطص هذا النظام ويتم تطـوير فكـرة دالة Impulse Response لنظام العتبة . وتُقترح نماذج ريـاضية غير خطية لنمذجـة ديناميكية السفينـة . ويتم تبيان أن هذه النماذج تُجهـز ملاطمة أفضل وتعطي تكلفة أقل من النماذج الخطيـة في تصميم مُسيطر أمثل .

*Key Words* : Threshold System, Ship's System, Impulse Response Function, Controller.

## ABSTRACT

In this paper we consider an application of a system called threshold system in the field of marine engineering. Properties of this system are discussed and the idea of the impulse response function of threshold system is developed. Non-linear models are suggested for modelling ship's dynamics. It is shown that these non-linear models provide better fit and give lower cost than linear models in designing an optimal controller.

## INTRODUCTION

Modern control theory requires a precise description of system dynamics by a mathematical model. To provide a dynamic description of the ship's motion as well as a proper criterion function of the performance of the steering law, two approaches to this problem may be considered. The first is deterministic and based on the well known first or second order differential equation of ship's manouverability. The second is statistical and regards ship's behaviour as a stationary time series.

Ship's system has two types of variables [see Fig. (1)]: Controlled variables (Yawing, rolling, pitching, etc.) and a control variable (rudder angle). The idea of "threshold" is reasonable for such a system. The simplest type of automatic rudder control instruments gives one of two command signals for the position $\psi$, namely $\psi = \pm \psi_o$. The response of the ship's orientation to $\psi$ as measured by the angle Ø satisfies (under appropriate conditions)

$$I \frac{d^2\varphi}{dt^2} + H \frac{d\varphi}{dt} = M(\psi),$$

where I and H are parameters of the ship's rudder, M $(\psi_o) = M_o$ and $M(-\psi_o) = -M_0$. Through the dependence of $\psi$ on Ø and dØ/dt, an appropriate equation of the following form may be derived (Andronov et al. [1]):

$$\begin{aligned} M(\psi) &= M[\psi(\varphi, d\varphi/dt)] \\ &= M_o Z(\varphi + b\, d\varphi/dt), \end{aligned}$$

where b is a constant and Z ( . ) is defined by

$$Z(x) = \begin{cases} +1 \ ; & x \le 0 \\ -1 \ ; & x > 0 \end{cases}$$

The object here is to investigate operating conditions under which there is no limit cycle.

Ohtsu et al. [3] represented the actual ship's course keeping motion by an autoregressive (AR) model. Ohtsu et al. [4] described an approach to the optimal controlling ship's course keeping motion by an AR model and they used the identified model to design an optimal controller (AR autopilot). In this paper we try to use non-linear stochastic models called threshold autoregressive (TAR) models in the modelling of ship's dynamics.

## THRESHOLD SYSTEM

Consider a signle input and a single output system in which R ( t ) denotes the input, X ( t ) the true (unobservable) output, N ( t ) an additive noise disturbance, and Y ( t ) is the observed ouput. If we assume that the system is linear, i.e. the present output is a fixed linear combination of present and past inputs for all t, then the relationships between R ( t ), X ( t ), N ( t ) and Y(t) may be expressed as (see; e.g. Priestley [ 5 ] )

$$X(t) = \sum_{i=0}^{\infty} c(i)\ R(t-i), \qquad (2.1a)$$

$$Y(t) = X(t) + N(t), \qquad (2.1b)$$

where the sequence of constants { c ( i ) } is the impulse response function of the system.

Suppose now that the system is non-linear and assume that it consists of a finite number of subsystems, and one and only one subsystem is employed at each instant of time. This system is called threshold system. Suppose also that there is an indicator variable J ( t ) that indicates which subsystem is to be employed at each instant. J ( t ) is an integer valued variable which takes the values 1, 2, ... , L; for some positive integer L.

We can represent an open-loop system by a threshold autoregressive model of the form

$$Y(t) = a_0^{(j)} + \sum_{i=0}^{q_j} b_i^{(j)} R(t-i) + N(t)^{(j)}; \qquad (2.2)$$

for some integer $q_j$ conditional on J ( t ) = j ; j = 1, 2, ... , L. { N ( t ) ( j ) } are white noise sequences each with zero mean and finite variance and each being independent of R ( t ). These L sequences are assumed to be pairwise independent.

Similarly, the closed -loop threshold system is represented as

$$Y(t) = a_0^{(j)} + \sum_{i=0}^{q_j} b_i^{(j)} R(t-i) + N(t)^{(j)}; \qquad (2.3a)$$

for some integer $q_j$ conditional on $J_1(t) = j$ ; j = 1, 2 ..., $L_1$ , and

$$R(t) = c_0^{(j)} + \sum_{i=0}^{s_j} d_i^{(j)}\ Y(t-i) + e(t)^{(j)}; \qquad (2.3b)$$

for some integer $s_j$ conditional on $J_2(t) = j$ ; j = 1, 2 ..., $L_2$ . $J_1(t)$ and $J_2(t)$ being the indicator variables for each of the two loops and { $e(t)^{(j)}$ } is the white noise sequence of the second loop. This sequence has similar properties of { $N(t)^{(j)}$ }.

From the statistical point of view past output contains some information about present output. Hence, the representation (2.2) may be generalized as

$$Y(t) = a_0^{(j)} + \sum_{i=1}^{p_j} a_i^{(j)}\ Y(t-i) + \sum_{i=0}^{q_j} b_i^{(j)} R(t-i) + N(t)^{(j)} \qquad (2.4)$$

for some integers $p_j$ and $q_j$ . The generalizations of (2.3) are respectively

$$Y(t) = a_o^{(j)} + \sum_{i=1}^{p_j} a_i^{(j)} Y(t-i) + \sum_{i=0}^{q_j} b_i^{(j)} R(t-i) + N(t)^{(j)} \qquad (2.5a)$$

and

$$R(t) = c_0^{(j)} + \sum_{i=1}^{r_j} c_i^{(j)} R(t-i) + \sum_{i=0}^{s_j} d_i^{(j)} Y(t-i) + e(t)^{(j)} \qquad (2.5b)$$

for some integers $p_j$ , $q_j$ , $r_j$ , $s_j$ , The condition stated for equations (2.2) and (2.3) still stand completely for equations (2.4) and (2.5).

Given the input and the output from a real system, if we want to fit a model of one of the above forms to the data, the unknown regression parameters can be estimated by using the well-known least squares method. Orders of difference equations and some other parameters can

be estimated by using the Akake's information criterion (see Sakamoto [6]) which is denoted by AIC. The normalized AIC is NAIC (k) = $n^{-1}$ (n in (Residual Variance) + 2 k ), (2.6)
where n is the sample size. The appropriate model is determined by the value of k at which NAIC (k) attains its minimum value.

After fitting a model to some observations it is important to test whether the residuals of the model satisfy the two usual assumptions of independency and normality. The procedure for testing these two assumptions can be summarized as follows.

i. If not less than 95% of the autocorrelations of the residuals of lags 1,2, ...., 100 lie within the band $\pm 1.96/n^{1/2}$, then the residuals are accepted as white noise or uncorrelated.

ii. The Z statistic of Lin and Mudholkar [2] is then used to test for normality. This statistic is asymptotically normal with zero mean and unit variance, under the null hypothesis of normal residuals.

| Variable | Mean | Variance | Range |
|---|---|---|---|
| R ( t ) | 601.9 | 53155 | [72,952] |
| Y ( t ) | – 28.8 | 4728 | [– 372,136] |

Bivariate histograms of R ( t ) and Y ( t ± i ) ; i = 0 , 1 ; are shown in Fig. (2). From this figure we note that these distributions are non-Gaussian with multimodes.

Esimates of the regression function of R ( t ) on Y (t–i), E (R ( t ) / Y (t–i) ) i = 0, 1; are shown in Fig. (3), while these of Y ( t ) on R (t–i); i = 0, 1; are shown in Fig. (4). The obvious feature of these figures is the non - linear shape of these regressions functions. An analysis of the regression function of these variables based on the bivariate index of linearity of Thanoon et al. [7] indicated that there is a delay time between R (t) and Y (t) of 3 time units.

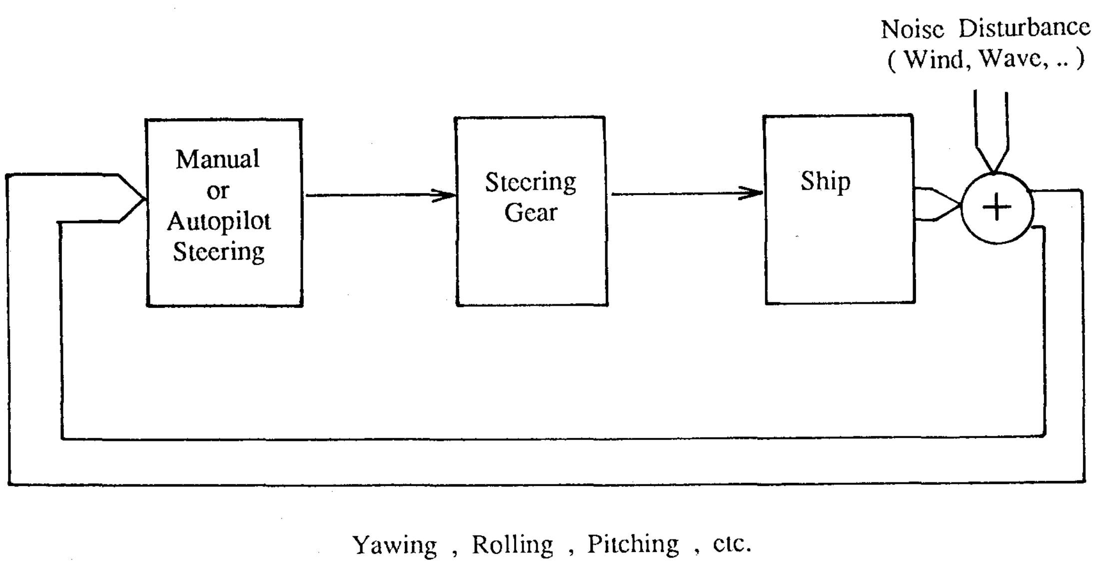


Fig 1
SHIP'S SYSTEM

The date of this study is chosen from the Computer Science Monograph (No. 11, TIMSAC–78) published by the Institute of Statistica Mathematics, Tokyo, Japan. The control variable is rudder angle, R (t), and the controlled variable is yawing, Y (t). We consider the first 250 data points in the monograph. For this data we have the following information :

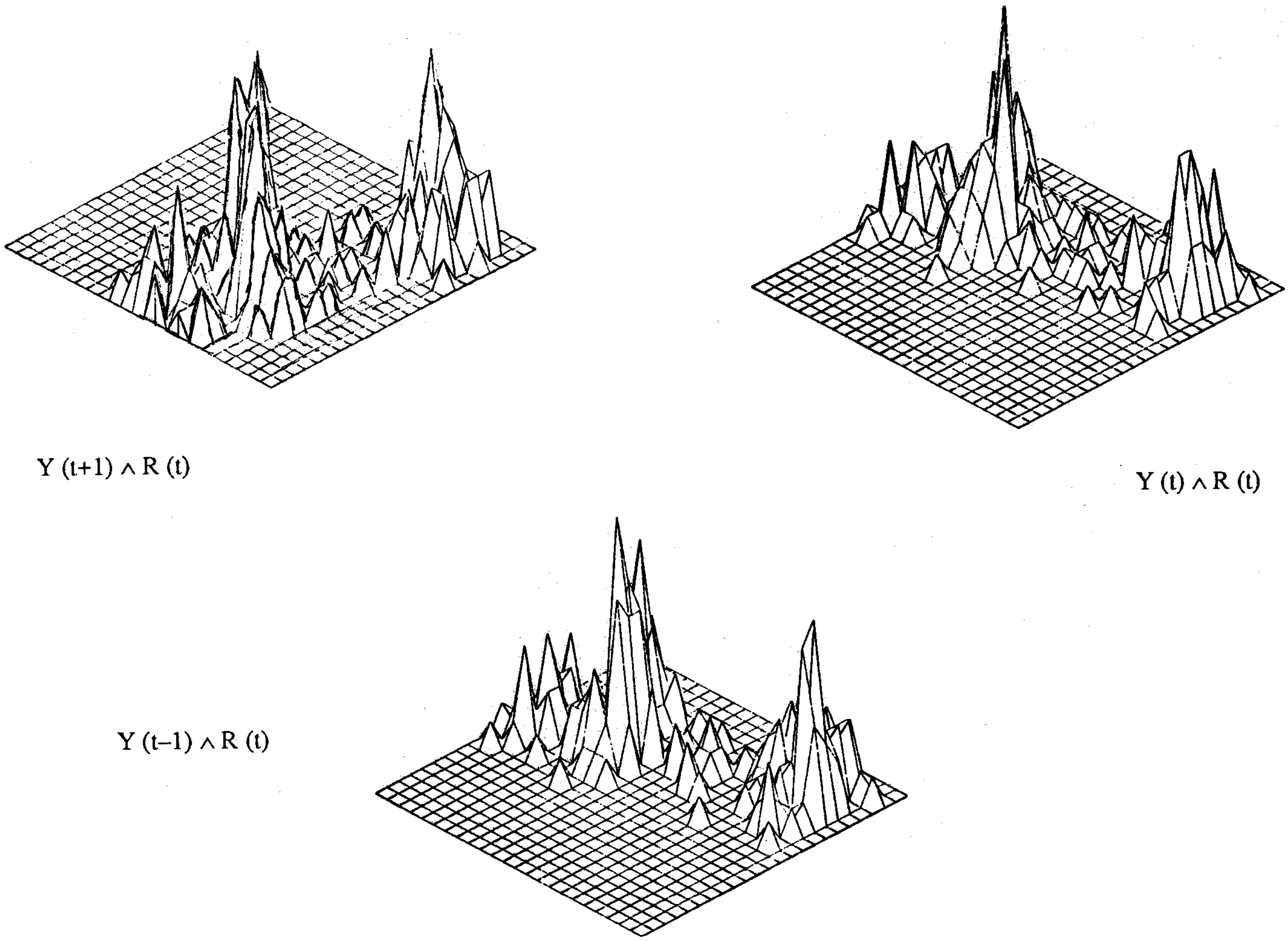


Fig. 2
Bivariate Histograms of R (t) and Y (t±1) for Raw Data

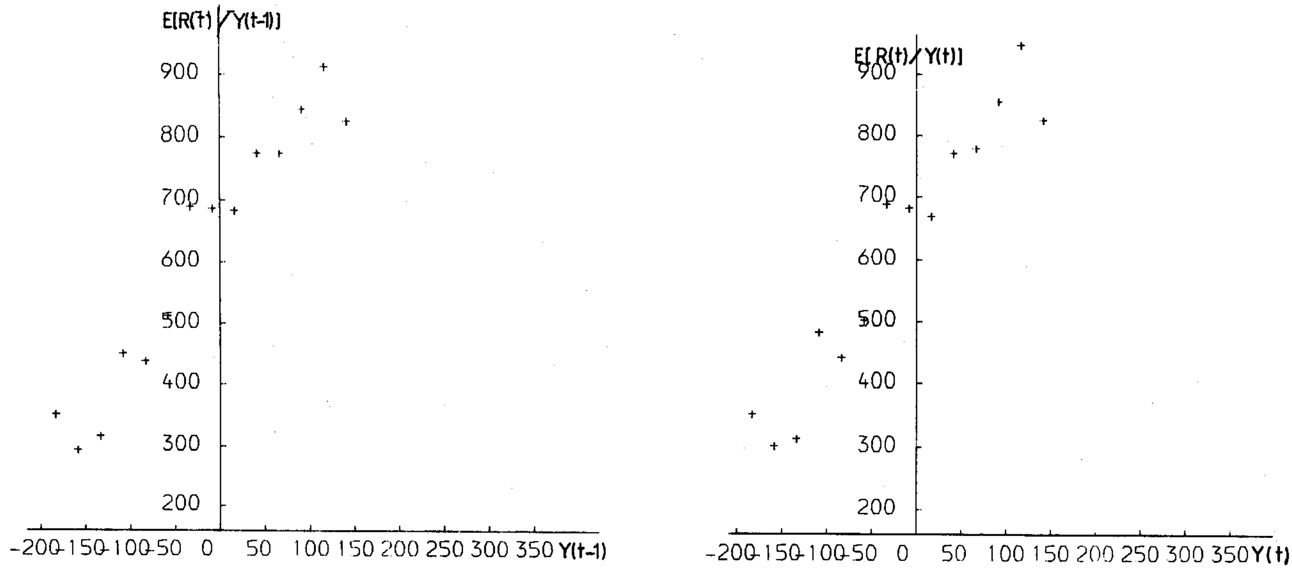


Fig. 3
Regression Function of R(t) on Y (t-1) for Raw Data.

## MODELLING SHIP'S SYSTEM

If we consider ship's system a linear system, the following linear model is obtained for the first loop.

$$
\begin{aligned}
Y(t) = & -\underset{(0.5)}{6.91} - \underset{(0.07)}{0.62}\,Y(t-1) + \underset{(0.08)}{0.16}\,Y(t-2) \\
& + \underset{(0.08)}{0.02}\,Y(t-3) - \underset{(0.08)}{0.12}\,Y(t-4) \\
& - \underset{(0.08)}{0.002}\,Y(t-5) + \underset{(0.08)}{0.18}\,Y(t-6) + \underset{(0.07)}{0.04}\,Y(t-7) \\
& - \underset{(0.05)}{0.04}\,R(t) + \underset{(0.07)}{0.09}\,R(t-1) \\
& - \underset{(0.07)}{0.06}\,R(t-2) + \underset{(0.08)}{0.08}\,R(t-3) + \underset{(0.07)}{0.02}\,R(t-4) \\
& - \underset{(0.05)}{0.08}\,R(t-5) + N(t), \qquad (4.1a)
\end{aligned}
$$

where the estimated residual variance is Var (N ( t ) ) = 1308.9 and NAIC = 7.30. The bracketed entries in (4. la) are the approximate standard errors of the extimated paramaters. For the second loop the following linear model is identified.

$$
\begin{aligned}
R(t) = & \ \underset{(64.03)}{64.17} + \underset{(0.07)}{1.01}\,R(t-1) - \underset{(0.10)}{0.01}\,R(t-2) \\
& - \underset{(0.09)}{0.06}\,R(t-3) - \underset{(0.06)}{0.03}\,R(t-4) \\
& - \underset{(0.08)}{0.26}\,Y(t) + \underset{(0.10)}{0.40}\,Y(t-1) \\
& + \underset{(0.09)}{0.18}\,Y(t-2) + e(t), \qquad (4.1b)
\end{aligned}
$$

where Var (e ( t ) ) = 1678.4 and NAIC = 7.49

We consider now ship's system as a non-linear system. For the first loop the following indicator variable is identified by using the NAIC procedure (see next section).

$$
J_1(t) = \begin{cases} 1 & \text{if } R(t-3) \le 400 \\ 2 & \text{if } R(t-3) > 400 \end{cases} \qquad (4.2a)
$$

$$
Y(t) = \begin{cases}
\underset{(0.11)}{1.11}\,Y(t-1) - \underset{(0.10)}{0.29}\,Y(t-2) - \underset{(0.09)}{0.15}\,R(t) \\
\quad + \underset{(0.10)}{0.10}\,R(t-1) + N_1(t) & \text{if } J_1(t) = 1 \\[2ex]
\underset{(0.09)}{0.47}\,Y(t-1) + \underset{(0.10)}{0.26}\,Y(t-2) + \underset{(0.11)}{0.08}\,Y(t-3) \\
\quad - \underset{(0.11)}{0.12}\,Y(t-4) + \underset{(0.11)}{0.04}\,(t-5) \\
\quad + \underset{(0.10)}{0.38}\,Y(t-6) - \underset{(0.08)}{0.05}\,Y(t-7) - \underset{(0.08)}{0.02}\,R(t) \\
\quad + \underset{(0.11)}{0.09}\,R(t-1) - \underset{(0.11)}{0.12}\,R(t-2) \\
\quad + \underset{(0.11)}{0.04}\,R(t-3) + \underset{(0.11)}{0.24}\,R(t-4) - \underset{(0.08)}{0.23}\,R(t-5) \\
\quad + N_2(t) & \text{if } J_1(t) = 2
\end{cases} \qquad (4.3a)
$$

where Var ($N_1$ (t)) = 800.3, Var ($N_2$ (t)) = 1343.0 , the pooled residual variance is 1166.6 and NAIC = 7.18.

For the second loop, the following indicator variable is obtained

$$
J_2(t) = \begin{cases} 1 & \text{if } |Y(t-3)| \le 19 \\ 2 & \text{if } |Y(t-3)| > 19 \end{cases} \qquad (4.2b)
$$

The following TAR model is then obtained

$$
R(t) = \begin{cases}
\underset{(34.81)}{272.7} + \underset{(0.13)}{0.54}\,R(t-1) + \underset{(0.17)}{0.57}\,R(t-2) \\
\quad - \underset{(0.18)}{0.73}\,R(t-3) + \underset{(0.20)}{0.41}\,R(t-4) \\
\quad - \underset{(0.17)}{0.37}\,R(t-5) + \underset{(0.19)}{0.29}\,R(t-6) - \underset{(0.20)}{0.13}\,R(t-7) \\
\quad + \underset{(0.15)}{0.12}\,R(t-8) - \underset{(0.14)}{0.13}\,R(t-9) \\
\quad + \underset{(0.09)}{0.06}\,R(t-10) - \underset{(0.12)}{0.30}\,Y(t) + \underset{(0.14)}{0.04}\,Y(t-1) \\
\quad - \underset{(0.55)}{0.66}\,Y(t-2) - \underset{(0.18)}{0.37}\,Y(t-3) \\
\quad + \underset{(0.16)}{0.35}\,Y(t-4) - \underset{(0.19)}{0.10}\,Y(t-5) + \underset{(0.19)}{0.24}\,Y(t-6) \\
\quad - \underset{(0.17)}{0.28}\,Y(t-7) + \underset{(0.15)}{0.49}\,Y(t-8) \\
\quad + e_1(t) & \text{if } J_2(t) = 1 \\[2ex]
\underset{(12.06)}{51.42} + \underset{(0.07)}{1.09}\,R(t-1) - \underset{(0.10)}{0.17}\,R(t-2) \\
\quad - \underset{(0.09)}{0.25}\,Y(t) + \underset{(0.09)}{0.53}\,Y(t-1) + e_2(t) & \text{if } J_2(t) = 2
\end{cases} \qquad (4.3b)
$$

where Var ($e_1$ (t)) = 547.6 , Var ($e_2$ (t)) = 1662.0, the pooled residual variance is 1420.5 and NAIC = 7.39.

Let us now try to test the residuals of the above models. Table I shows the residual variance and the NAIC values of each model. The fourth column gives the percentage of autocorrelations of residuals which lie inside the band $\pm 1.96\ n^{1\backslash 2}$. The last column shows the value of the Z-statistic for testing the normality of these residuals. From this table we conclude that the residuals of all these models can be accepted as Guassian white noise. It is also clear from the residual variance as well as the NAIC values that the TAR models provide better fit than the corresponding linear models.

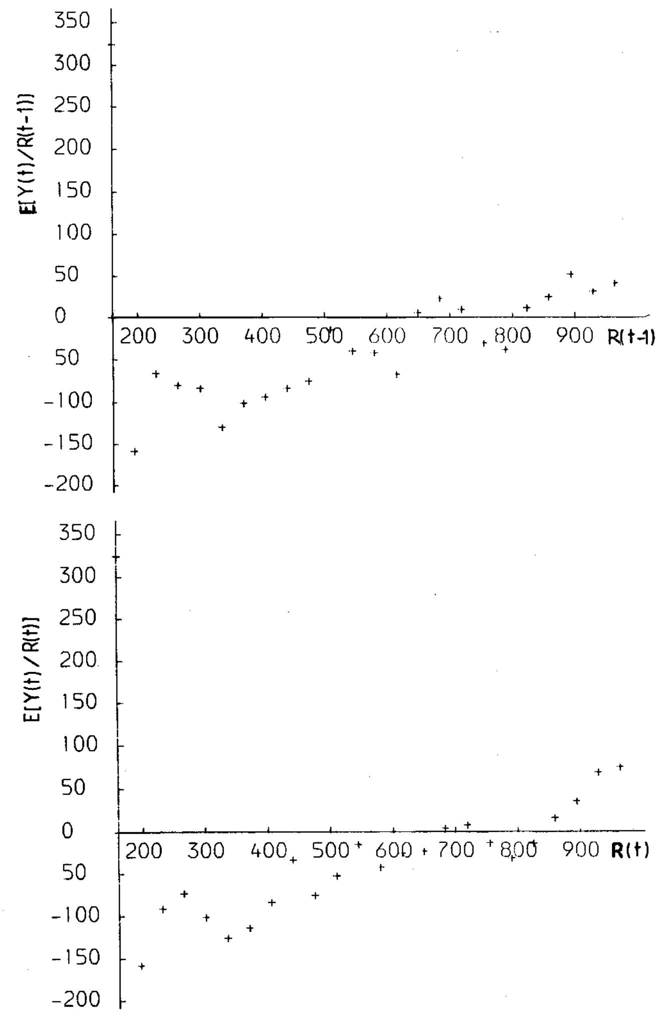


Fig. 4
Regression Function of Y (t) on R ( t - 1 ) for Raw Data.

## THE IMPULSE RESPONSE FUNCTION

Let us consider ship's system as a single input and a single output system in which R (t) denotes the input, X (t) the true (unobservable) output, N (t) an additive noise disturbance, and Y (t) is the observed output. The relationship between these variables may be expressed as

$$X(t) = g\,(R(t), R(t-1), \ldots) \tag{5.1a}$$
$$Y(t) = X(t) + N(t), \tag{5.1b}$$

where g (.) is an unknown function. If we assume that g (.) is linear and time invariant, we can rewrite (5.1a) as

$$X(t) = \sum_{i=0}^{\infty} c(i)\ R(t-i), \tag{5.2}$$

**Table I**

A Comparison between the Fitted Models

| Model | Residual Variance | NAIC | Whiteness Test ( % ) | Z – Statistic |
|---|---|---|---|---|
| ( 4.1a ) | 1308.9 | 7.30 | 98 | 0.90 |
| ( 4.1b ) | 1678.4 | 7.49 | 95 | 1.37 |
| ( 4.3a ) | 1166.6 | 7.18 | 99 | 1.94 |
| ( 4.3b ) | 1420.5 | 7.39 | 96 | 0.69 |

where the sequence of constants { c ( i ) } is the impulse response function.

We defined in section 2 threshold system to be that system which consists of a finite number of subsystems, and one and only one subsystel is employed at each instant of time. Also, we defined J (t) to be an indicator variable which indicates which subsystem is employed at time t. Typically, J (t) is defined in terms of past input R (t–d), for some positive integer d. Then for simplicity of discussion we consider the following simple TAR model after ignoring the intercept and the noise term

$$Y(t) = a^{(J(t))}\, Y(t-1) + b^{(J(t))}\, R(t). \tag{5.3}$$

To obtain an expression for the output as a function of present and past input, we need to expand Y (t–1) in the right hand side of (5.3) infinite number of times. Now, if (5.3) is a linear model. i.e. J (t) = m, a constant for all t. Then if $|a^{(m)}| < 1$, there is a unique set of coefficients of R (t), R (t-1) such that we can write (5.3) as

$$Y(t) = \sum_{i=0}^{\infty} C^{(m)}(i)\ R(t-i), \tag{5.4}$$

where $C^{(m)}(i) = b^{(m)} a^{(m)^i}$ . Now consider the case when J (t) is not a constant. Since J (t) depends on R (t–d) and $a^{(J(t))}$, $b^{(J(t))}$, depend on J (t), it is clear that the invertibility of (5.3) leads to infinite number of sets of coefficients of R (t), R (t–1), .... : where each set depends on present and past values of the indicator variable J (t). i.e.

$$Y(t) = b^{(J(t))} R(t) + a^{J(t))} b^{(J(t-1))} R(t-1)$$

$$+ a^{(J(t))} b^{(J(t-1))} b^{(J(t-2))} R(t-2) + ...$$

or

$$Y(t) = \sum_{i=0}^{\infty} C^{(J(t))}(i) R(t-i), \qquad (5.5a)$$

where

$$C^{(J(t))}(i) = a^{(J(t))} \prod_{j=1}^{i} b^{(J(t-j))} \; ; i = 1,2,... \qquad (5.5b)$$

and { $C^{(J(t))}(i)$ } is the impulse response function of this threshold system. Note that for $C^{(J(t))}(i)$ to converge, we need to assume that | b (j) | < 1 for all j = 1,2,..., L. This form of the impulse response function of treshold system is interesting because it covers all possibilities of behaviour of the L subsystems in present and past.

Let us now apply the above ideas on the fitted models in the last section. After inverting the linear model (4. 1a) we get

$$Y(t) = -5.08 + \sum_{i=0}^{\infty} C_1(i) \; R(t-i), \qquad (5.6)$$

where the plot of the impulse response function $\{C_1(i)\}$ is shown in Fig. (5). Inverting the linear model (4.1b) gives

$$R(t) = 706.0 + \sum_{i=0}^{\infty} C_2(i) \; Y(t-i), \qquad (5.7)$$

Let us now consider the TAR model (4.3a). As we have mentioned before there is an infinite number of sets of coefficients of R (t), R (t–1),... depending on the values of $J_1$ (t), $J_1$ (t–1),.... . We consider here two special cases :

(i) If $J_1(t) = J_1(t-1) = ... = 1$ (i.e. always rudder angle does not exceed 400), then

$$Y(t) = \sum_{i=0}^{\infty} C^{(1)}(i) R(t-i), \qquad (5.8a)$$

where { $C^{(1)}(i)$ } is shown in Fig. (7a).

(ii) If $J_1(t) = J_1(t-1) = .. = 2$ (i.e. always rudder angle exceeds 400), then

$$Y(t) = \sum_{i=0}^{\infty} C^{(2)}(i) R(t-i), \qquad (5.8b)$$

where $\{C^{(2)}(i)\}$ is shown in Fig. (7b).

Similarly we consider two cases for the TAR model (4.3b) :

(i) If $J_2(t) = J_2(t-1) = ... = 1$, we have

$$R(t) = 7.8 \times 10^{36} + \sum_{i=0}^{\infty} C^{(3)}(i) Y(t-i), \qquad (5.9a)$$

(ii) If $J_2(t) = J_2(t-1) = ... = 2$, we have

$$R(t) = 700.6 + \sum_{i=0}^{\infty} C^{(4)}(i) Y(t-i), \qquad (5.9b)$$

where $\{C^{(3)}(i)\}$ , $\{C^{(4)}(i)\}$ are shown in Fig. (8). Note that $\{C^{(3)}(i)\}$ is divergent because some of the roots of the characteristic equation $Z^{10} - 0.54Z^9 - 0.57Z^8 + 0.73Z^7 - 0.41Z^6 + 0.37Z^5 - 0.29Z^4 + 0.13Z^3 - 0.12Z^2 + 0.13Z - 0.06 = 0$ lie outside the unit circle.

## MINIMUM VARIANCE CONTROL

Suppose we wish to construct a feedback controller to compensate for the disturbance N (t) so that the output Y (t) follows, as far as possible, some predetermined form. Without loss of generality we may take the original predetermined form to be zero, for all t, (which is then called the "set point") and the problem now is to design a controller which, for each t, computes R(t) as a function F (.) of zero. The optimal form of F ( . ) is then determined by minimizing some "cost function" V which measures the cost of deviations of Y (t) from its set point zero. In minimum variance control we choose as our cost function the expected value of $Y^2(t+d)$. i.e.

$$V = E\,[\,Y^2(t+d)], \qquad (6.1)$$

where d is the delay time between input and output. Then we choose F ( . ) to minimize this form of V. It can be shown that, see e.g. Priestley [5], the optimal choice of the input R (t) is that for which the true output X (t) satisfies

$$X(t+d) = -\hat{N}(t+d), \qquad (6.2)$$

where $\hat{N}(t+d)$ is the d-step-ahead predictor of N (t+d). From (5.1b), (6.1) and (6.2) we get

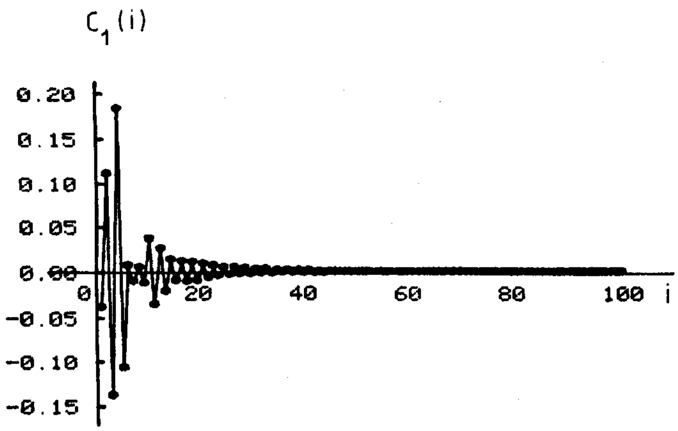


Fig. (5) : Impulse Response Function of (4.1a)

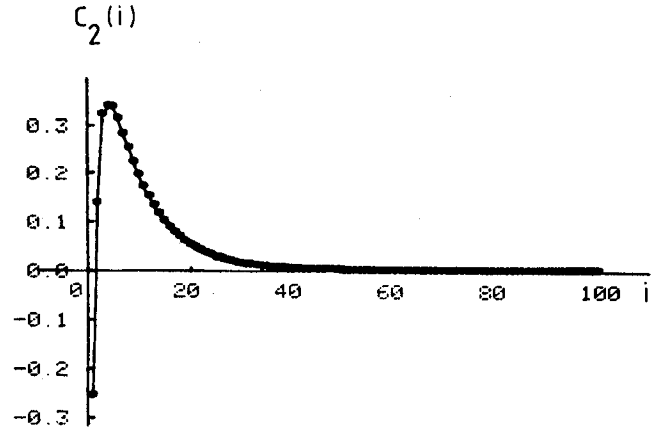


Fig. (6) : Impulse Response Function of (4.1b)

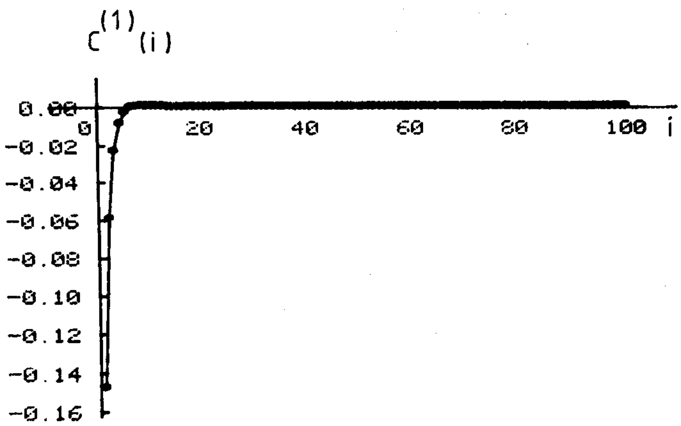


Fig. (7a) Impulse Response Function of (4.3a) when $J_1(t) = J_1(t-1) = .... = 1$

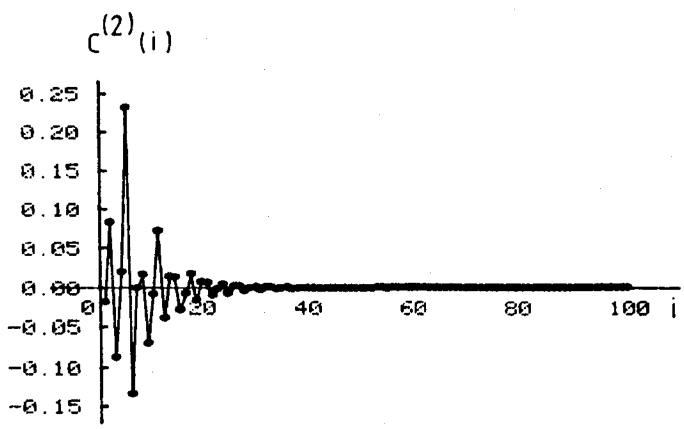


Fig. (7b) Impulse Response Function of (4.3a) when $J_1(t) = J_1(t-1) = .... = 2$

$$V = E [ \{ N(t+d) - \hat{N}(t+d) \} ]^2 \qquad (6.3)$$

or

V = mean square error (m.s.e.) of the d-step-predictor of N (t+d).

Ohtsu et al. [4] used an identified AR model to design an optimal controller. Here we have no actual ship to design such controller. What we have to do is to consider the sample m.s.e. as our cost function.

Table II gives the sample m.s.e. of h-step-ahead predictors from the fitted models for rudder and yawing for the last 70 data point, h = 1,2,3. From the table we note that TAR models give lower cost function than linear AR models.

**Table II**

m.s.e. of h-step-ahead predictors

| Model \ h | 1 | 2 | 3 |
|---|---|---|---|
| ( 4.1a ) | 113906 | 17609 | 95543 |
| ( 4.1b ) | 2841 | 8724 | 17556 |
| ( 4.3a ) | 2303 | 6084 | 11063 |
| ( 4.3b ) | 2777 | 8742 | 17161 |

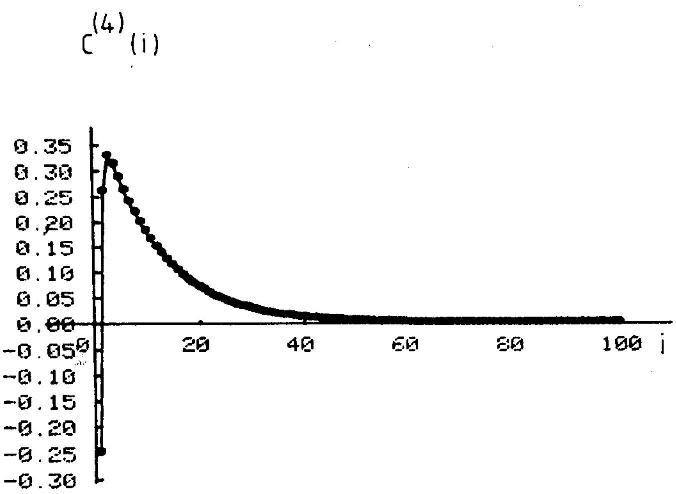


Fig. (8a) Impulse Response Function of (4.3b) when $J_2(t) = J_2(t-1) = .... = 1$

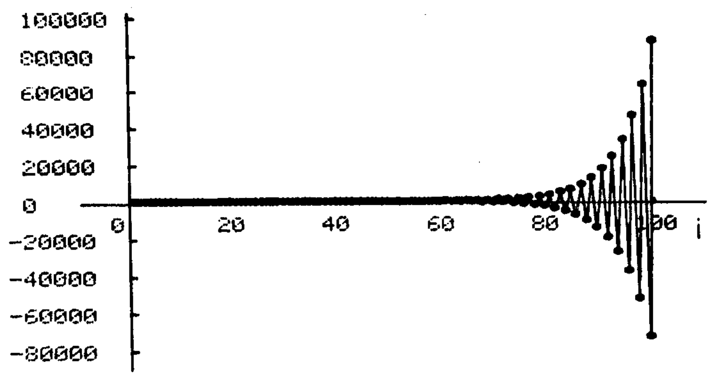


Fig. (8b) Impulse Response Function of (4.3b) when $J_2(t) = J_2(t-1) = .... = 2$

## CONCLUSION AND DISCUSSION

The idea of threshold system seems to be reasonable in applications. We gave the indicator process of threshold system some physical meaning where rudder angle was classified in to two states with respect to ship's motion: one when there is no correction to yawing angle and the other when there is.

The fitted threshold models characterize by good statistical and fitted threshold models have no limit cycles. In fact the threshold model for yawing (4.3a) gives a limit point of zero, while the threshold model for rudder angle (4.3b) gives a limit point of 352.2.

In order to compare the linear system, which consists of equations (4.1a) and (4.1b), with threshold system, which consists of equations (4.3a) and (4.3b), simulation based on 6000 data points of these systems was carried out. Figs. (9), (10) show the obtained bivariate distributions from these systems, while Figs. (11) to (14) show the estimated regression functions from close to these of the real data, shown in Fig. (2) to (4), than these from linear system.

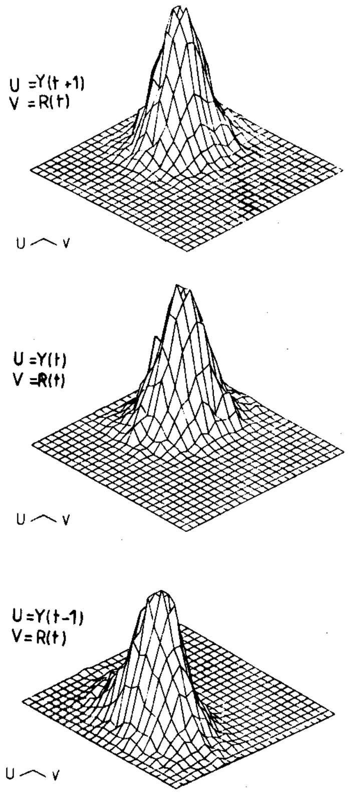


Fig. (9) : Bivariare Histograms of R (t) and Y (t ± i) from Linear System

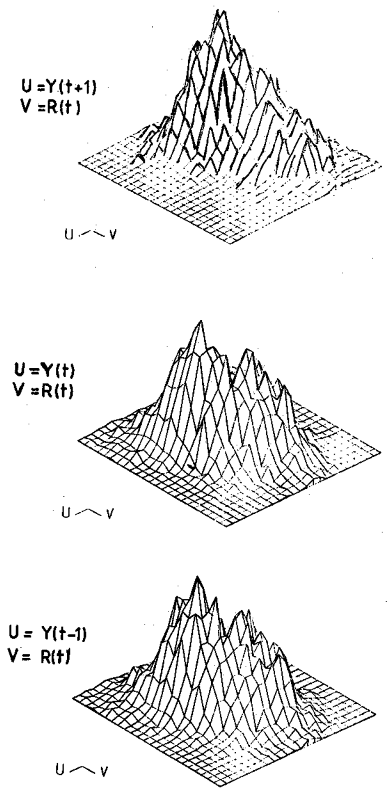


Fig. (10) : Bivariate Histograms of R (t) and Y (t ± i) from Threshold System

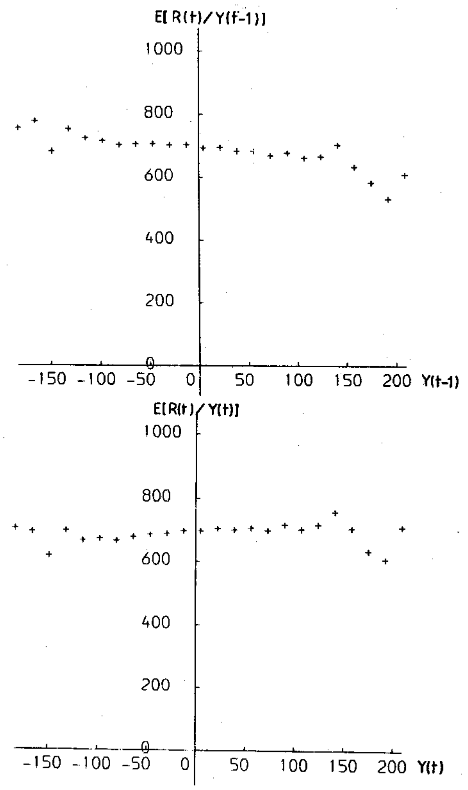


Fig. (11) : Regression Function of R on Y ( t - i ) for linear system

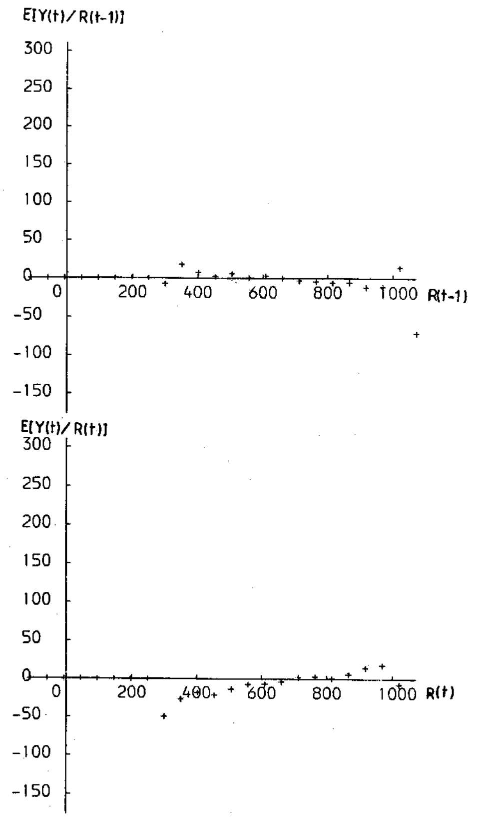


Fig. (12) : Regression Function of Y (t) on R (t – i) for Linear System

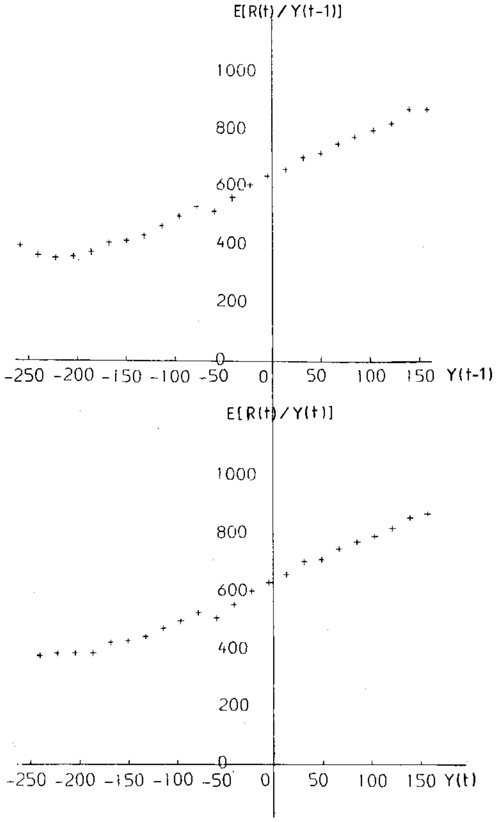


Fig. (13) : Regression Function of R (t) on Y (t – i) for Threshold System

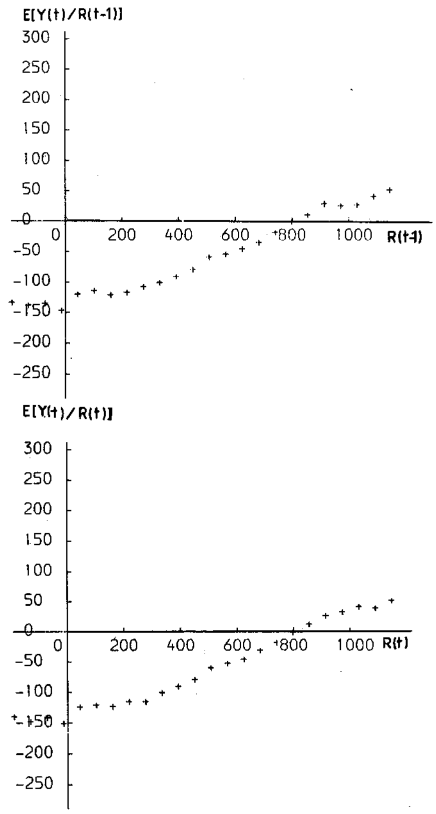


Fig. (14) : Regression Function of Y (t) on R (t – i) for Threshold System

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