New solitary wave and multiple soliton solutions for fifth order nonlinear evolution equation with time variable coefficients

H.M. Jaradat [a], Muhammed Syam [b], M.M.M. Jaradat [c,*], Zead Mustafa [c], S. Momani [d,e]

[a] Department of Mathematics, Al al-Bayt University, Mafraq, Jordan
[b] Department of Mathematical Sciences, United Arab Emirates University, Al Ain, United Arab Emirates
[c] Department of Mathematics, Statistics and Physics, Qatar University, Doha, Qatar
[d] Department of Mathematics, Faculty of Science, The University of Jordan, Amman 11942, Jordan
[e] Nonlinear Analysis and Applied Mathematics Research Group, Faculty of Science, King Abdulaziz University, Saudi Arabia

ARTICLE INFO

ABSTRACT

In this paper, we investigate the multiple soliton solutions and multiple singular soliton solutions of a class of the fifth order nonlinear evolution equation with variable coefficients of t using the simplified bilinear method based on a transformation method combined with the Hirota's bilinear sense. In addition, we present analysis for some parameters such as the soliton amplitude and the characteristic line. Several equation in the literature are special cases of the class which we discuss such as Caudrey-Dodd-Gibbon equation and Sawada-Kotera. Comparison with several methods in the literature, such as Helmholtz solution of the inverse variational problem, rational exponential function method, tanh method, homotopy perturbation method, exp-function method, and coth method, are made. From these comparisons, we conclude that the proposed method is efficient and our solutions are correct. It is worth mention that the proposed solution can solve many physical problems.

© 2018 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

Introduction

In recent years studies on fifth-order nonlinear evolution equations have received considerable attention because these equations have many applications in physics such as nonlinear optics and Bose-Einstein. Choudhuri et al. [23] studies the following class of equations

$$w_t + aww_{xxx} + bw_xw_{xx} + cw^2w_x + w_{xxxxx} = 0 \qquad (1.1)$$

where $a, b,$ and c are constants. They used the Helmholtz solution of the inverse variational problem to derive its solution. conditions under which this equation admits an analytic representation. They studied only the case when $A = a(t), B = b(t),$ and $C = c(t)$ are constants. Mia et al. [18] discussed the Caudrey-Dodd-Gibbon equation (CDGE) of the form

$$w_t + 30ww_{xxx} + 30w_xw_{xx} + 180w^2w_x + w_{xxxxx} = 0 \qquad (1.2)$$

by tanh-method. Wazwaz [19] solved the solved the same equation using the coth-method. Abdollahzadeh et al. [20] solved (CDGE) using the rational exponential function method. Ghasemi et al. [21] used the homotopy perturbation method for solving the Sawada-Kotera (SK) Equation

$$w_t + 15ww_{xxx} + 15w_xw_{xx} + 45w^2w_x + w_{xxxxx} = 0. \qquad (1.3)$$

Naher et al. [22] studied the solution of the SK Eq. (4.2) using the exp-function method. In this paper, we consider the following class of the fifth order nonlinear evolution equation (FNEE) with variable coefficients of t:

$$w_t + a(t)ww_{xxx} + b(t)w_xw_{xx} + c(t)w^2w_x + w_{xxxxx} = 0 \qquad (1.4)$$

where $w = w(x, t)$ and the coefficients $a(t), b(t)$ and $c(t)$ are function of the variable t. Eqs. (1.1)–(1.3) are special cases of Problem (1.4). Thus, we compare our results with the solutions of Choudhuri et al. [23], Mia et al. [18], Abdollahzadeh et al. [20], Ghasemi et al. [21],

and Naher et al. [22] and we find that the proposed method generate their solitons. For this reason, we consider their problems as special case of the problem under discussion. More details about this problem can be found in [4–7,12–14,8–10].

Bilinear forms is one of the tools that usually used to solve nonlinear problems. More details about this method can be found in [1–3]. In this paper, we study the multiple soliton solutions and multiple singular soliton solutions of Eq. (1.4) without using the bilinear forms. The proposed technique depends on the simplified bilinear method based on a transformation method combined with the Hirota's bilinear sense. In the literature, when (a, b, c) are $(30, 60, 270), (20, 40, 120)$ and $(10, 20, 30)$, Eq. (1.4) becomes the fifth order KdV equation (FKdV) which satisfy the conditions

$$2a(t) = b(t), \quad c(t) = \frac{3a^2(t)}{10}.$$

When $(a(t), b(t), c(t))$ are $(30, 30, 180), (5, 5, 5), (15, 15, 45)$ and $(-15, -15, 45)$, Eq. (1.4) is called the Sawada-Kotera equation (SK) which satisfy the conditions

$$a(t) = b(t), \quad c = \frac{a^2(t)}{5}.$$

In this paper, we find that the corresponding conditions to determine single soliton solution on Eq. (1.4) are

$$c(t) = \frac{a^2(t) + a(t)b(t)}{10},$$
$$a(t) \neq -b(t),$$
$$b(t) = -a(t) + \alpha.$$

We organize this paper as follows. In Section "Multiple soliton solutions for (FNEE)", we present the multiple soliton solutions for Eqs. (1.4) while in Section "Multiple singular soliton solutions for (FNEE)", we derive the multiple singular soliton solutions for the problem under discussion. In Section "Analysis of the parameters and discussion", we present some analysis of the parameters and we make discussion about our results. Finally we conclude with some comments in the last section.

Multiple soliton solutions for (FNEE)

In this section, we apply the simplified bilinear method to construct the soliton solutions of Eq. (1.4). Substituting

$$w(x, t) = e^{\epsilon_i}, \tag{2.1}$$
$$\epsilon_i(x, t) = r_i x - s_i(t),$$

into the linear terms of Eq. (1.4), we obtain

$$s_i(t) = \int_0^t r_i^5 d\tau. \tag{2.2}$$

Then, the dispersion relation is given by

$$\epsilon_i(x, t) = r_i x - \int_0^t r_i^5 d\tau. \tag{2.3}$$

Let

$$w(x, t) = \rho(\ln \delta(x, t))_{xx} \tag{2.4}$$

where the auxiliary function $\delta(x, t)$ of single soliton solutions is given by

$$\delta(x, t) = 1 + e^{\epsilon_1(x, t)} = 1 + e^{r_1 x - \int_0^t r_1^5 d\tau}. \tag{2.5}$$

Substituting Eqs. (2.4) and (2.5) into Eq. (1.4) and solving the new equation for ρ to obtain

$$\rho = \frac{60}{a(t) + b(t)} \tag{2.6}$$

subject to

$$c(t) = \frac{a^2(t) + a(t)b(t)}{10}, \quad a(t) \neq -b(t), \; b(t) = -a(t) + \alpha, \tag{2.7}$$

where α is arbitrary nonzero constant.

Now, we can obtain the single soliton solution by

$$w(x, t) = \frac{60}{a(t) + b(t)} r_1^2 \frac{e^{\epsilon_1(x, t)}}{(1 + e^{\epsilon_1(x, t)})^2}$$
$$= \frac{15}{a(t) + b(t)} r_1^2 \sec h^2 \left(\frac{\epsilon_1(x, t)}{2} \right). \tag{2.8}$$

Next, we are looking for two-soliton solutions. Let

$$\delta(x, t) = 1 + e^{r_1 x - \int_0^t r_1^5 d\tau} + e^{r_2 x - \int_0^t r_2^5 d\tau} + c_{12} e^{(r_1 + r_2) x - \int_0^t (r_1^5 + r_2^5) d\tau}. \tag{2.9}$$

Substituting Eqs. (2.9), (2.6), and (2.4) into Equation (1.4), we obtain the two solutions for the phase shift as

$$c_{12} = \begin{cases} \frac{r_1^4 - 3r_1 r_2^3 - 3r_1^3 r_2 + 4r_1^2 r_2^2 + r_2^4}{r_1^4 + 3r_1 r_2^3 + 3r_1^3 r_2 + 4r_1^2 r_2^2 + r_2^4}, & \text{if } b(t) = a(t), c(t) = \frac{a^2(t)}{5}, \\ \frac{r_1^2 - 2r_1 r_2 + r_2^2}{r_1^2 + 2r_1 r_2 + r_2^2}, & \text{if } b(t) = 2a(t), c(t) = \frac{3a^2(t)}{10} \end{cases}. \tag{2.10}$$

The two-soliton solutions of the (FNEE) Eq. (1.4) are given by

$$w(x, t) = \begin{cases} \frac{30}{a(t)} \times \left[\frac{r_1^2 e^{\epsilon_1} + r_2^2 e^{\epsilon_2} + c_{12}(r_1 + r_2)^2 e^{\epsilon_1 + \epsilon_2}}{1 + e^{\epsilon_1} + e^{\epsilon_2} + c_{12} e^{\epsilon_1 + \epsilon_2}} - \right. \\ \left. \left(\frac{r_1 e^{\epsilon_1} + r_2 e^{\epsilon_2} + c_{12}(r_1 + r_2) e^{\epsilon_1 + \epsilon_2}}{1 + e^{\epsilon_1} + e^{\epsilon_2} + c_{12} e^{\epsilon_1 + \epsilon_2}} \right)^2 \right], & \text{if } b(t) = a(t), c(t) = \frac{a^2(t)}{5}, \\ \frac{20}{a(t)} \times \left[\frac{r_1^2 e^{\epsilon_1} + r_2^2 e^{\epsilon_2} + c_{12}(r_1 + r_2)^2 e^{\epsilon_1 + \epsilon_2}}{1 + e^{\epsilon_1} + e^{\epsilon_2} + c_{12} e^{\epsilon_1 + \epsilon_2}} - \right. \\ \left. \left(\frac{r_1 e^{\epsilon_1} + r_2 e^{\epsilon_2} + c_{12}(r_1 + r_2) e^{\epsilon_1 + \epsilon_2}}{1 + e^{\epsilon_1} + e^{\epsilon_2} + c_{12} e^{\epsilon_1 + \epsilon_2}} \right)^2 \right], & \text{if } b(t) = 2a(t), c(t) = \frac{3a^2(t)}{10} \end{cases}.$$

The three-soliton solutions can be obtained by assuming

$$\delta(x, t) = 1 + e^{\epsilon_1} + e^{\epsilon_2} + e^{\epsilon_3} + c_{12} e^{\epsilon_1 + \epsilon_2} + c_{13} e^{\epsilon_1 + \epsilon_3} + c_{23} e^{\epsilon_2 + \epsilon_3} + c_{123} e^{\epsilon_1 + \epsilon_2 + \epsilon_3}, \tag{2.11}$$

where

$$c_{ij} = \begin{cases} \frac{r_i^4 - 3r_i r_j^3 - 3r_i^3 r_j + 4r_i^2 r_j^2 + r_j^4}{r_i^4 + 3r_i r_j^3 + 3r_i^3 r_j + 4r_i^2 r_j^2 + r_j^4}, & \text{if } b(t) = a(t), c(t) = \frac{a^2(t)}{5}, \\ \frac{r_i^2 - 2r_i r_j + r_j^2}{r_i^2 + 2r_i r_j + r_j^2}, & \text{if } b(t) = 2a(t), c(t) = \frac{3a^2(t)}{10} \end{cases}. \tag{2.12}$$

Substituting Eqs. (2.11), (2.6), and (2.4) into Equation (1.4), we get

$$c_{123} = c_{12} c_{13} c_{23}. \tag{2.13}$$

Then, the three-soliton solutions are obtained. Then Nsoliton solutions exist for any order $N \geqslant 4$ [15,16].

Multiple singular soliton solutions for (FNEE)

To obtain a single singular soliton solution, we assume that the auxiliary function $\delta(x, t)$ is given by

$$\delta(x, t) = 1 - e^{\epsilon_1(x, t)} = 1 - e^{r_1 x - \int_0^t r_1^5 d\tau}. \tag{3.1}$$

Substituting Eqs. (2.4) and (3.1) into Eq. (1.4) and solving the new equation for ρ, we get

$$\rho = \frac{60}{a(t) + b(t)} \tag{3.2}$$

subject to the same conditions in Eq. (2.7). Then, the single singular soliton solution for (FNEE) Eq. (1.4) is given by

$$w(x,t) = \frac{-60}{a(t)+b(t)} r_1^2 \frac{e^{\epsilon_1(x,t)}}{(1-e^{\epsilon_1(x,t)})^2}$$

$$= \frac{-15}{a(t)+b(t)} r_1^2 \csc h^2 \left(\frac{\epsilon_1(x,t)}{2}\right). \tag{3.3}$$

To determine the two singular soliton solutions explicitly, we substitute

$$\delta(x,t) = 1 - e^{r_1 x - \int_0^t r_1^5 d\tau} - e^{r_2 x - \int_0^t r_2^5 d\tau} + c_{12} e^{(r_1+r_2)x - \int_0^t (r_1^5 + r_2^5) d\tau}, \tag{3.4}$$

in Eq. (1.4). We obtain the phase shift c_{12} as in Equation (2.10). Following the same procedure as described before to obtain the two singular soliton solutions of Eq. (1.4). For the singular three-soliton solutions, let

$$\delta(x,t) = 1 - e^{\epsilon_1} - e^{\epsilon_2} + e^{\epsilon_3} + c_{12}e^{\epsilon_1+\epsilon_2} + c_{13}e^{\epsilon_1+\epsilon_3} + c_{23}e^{\epsilon_2+\epsilon_3}$$

$$+ c_{123}e^{\epsilon_1+\epsilon_2+\epsilon_3}, \tag{3.5}$$

where c_{ij} are defined in (2.12). Following the same procedure as described before, we obtain

$$c_{123} = a_{12}a_{13}a_{23}.$$

The singular three-soliton solution can be obtained by substituting Eqs. (3.5) and (3.2) into Eq. (2.4).

Analysis of the parameters and discussion

The soliton amplitude amp can be expressed from Eq. (2.8) as

$$\left| \frac{15}{a(t)+b(t)} r_1^2 \right|.$$

Hence, the soliton amplitude amp for fifth order KdV equation (FKdV) is given by a

$$amp = \left| \frac{5}{a(t)} r_1^2 \right|,$$

as well as the soliton amplitude amp for the Sawada-Kotera equation (SK) given by

$$amp = \left| \frac{15}{2a(t)} r_1^2 \right|.$$

The characteristic line [17,5,6,11] for each solitary wave can be defined by

$$x = r_i^4 t, \quad i = 1, 2, 3, \ldots. \tag{4.1}$$

The velocity of the wave at time t can be expressed as

$$v_i = r_i^4. \tag{4.2}$$

The soliton amplitude amp of Eq. (2.8) depends only on $a(t)$ and $b(t)$ while the soliton amplitude amp of fifth order KdV equation (FKdV) and Sawada-Kotera equation (SK) depends only on $a(t)$. In addition, the solitonic amplitude decreases while $a(t)$ is increasing. However, from Eq. (4.2), we note that the propagation velocity is independent on the coefficient parameters $a(t), b(t)$ and $c(t)$.

Choudhuri et al. [23] used the Helmholtz solution of the inverse variational problem to derive conditions under which this equation admits an analytic representation. They studied only the case when $A = a(t), B = b(t)$, and $C = c(t)$ are constants. They derived the conditions $B = 2A, C = \frac{3A^2}{10}$. In addition, they use the homogeneous balance method to derive the single soliton solution which is given by

$$w(x,t) = \frac{20}{A} \frac{c_0 c_1 \sqrt{v} e^{\sqrt[4]{v}(x+vt)}}{\left(c_0 + c_1 e^{\sqrt[4]{v}(x+vt)}\right)^2}.$$

Their results are special case of our results. If we choose $c_0 = c_1 = 1, v = r^4$, then their single soliton solution is the same single soliton solution which is given in Eq. (4.2).

The Caudrey-Dodd-Gibbon equation, which is given by

$$w_t + 30ww_{xxx} + 30w_x w_{xx} + 180w^2 w_x + w_{xxxxx} = 0, \tag{4.3}$$

was solved by Mia et al. [18] by tanh-method and the solution was given by

$$w(x,t) = k^2 - k^2 \tanh^2(kx - 16k^5 t). \tag{4.4}$$

They found that the necessary conditions are $a(t) = b(t) = 30$ and $c(t) = \frac{a^2(t)}{5} = 180$. If we choose $k = \frac{r_1}{2}$, the solution in Eq. (4.4) becomes

$$w(x,t) = \frac{r_1^2}{4} - \frac{r_1^2}{4}\tanh^2\left(\frac{r_1}{2}x - \frac{r_1^5}{2}t\right) = \frac{r_1^2}{4}\sec h^2\left(\frac{r_1}{2}x - \frac{r_1^5}{2}t\right)$$

which the same solution given in Eq. (4.2). This means that the proposed method is also solving Caudrey-Dodd-Gibbon equation.

Wazwaz [19] solved Eq. (4.3) by coth-method and his solution was given by

$$w(x,t) = -\mu^2 \csc h^2(\mu x - 16\mu^5 t).$$

If we choose $\eta = \frac{r_1}{2}$, his solution will be the same as well as the one in Eq. (3.3). Abdollahzadeh et al. [20] solved Eq. (4.3) using the rational exponential function method and he found that the solution was given by

$$w(x,t) = \frac{\alpha^2(\cosh(\alpha(x-\alpha^4 t)) + \sinh(\alpha(x-\alpha^4 t)))}{(1 + \cosh(\alpha(x-\alpha^4 t)) + \sinh(\alpha(x-\alpha^4 t)))^2}.$$

If we choose $\alpha = r_1^2$, their solution will be the same solution as the one given in Eq. (4.2). Ghasemi et al. [21] used the homotopy perturbation method for solving the Sawada-Kotera (SK) Equation

$$w_t + 15ww_{xxx} + 15w_x w_{xx} + 45w^2 w_x + w_{xxxxx} = 0. \tag{4.5}$$

and they found the solution was

$$w(x,t) = 2k^2 \sec h^2(k(x - 16k^4 t - x_0)).$$

If we choose $k = \frac{r_1}{2}$ and $x_0 = 0$, then their solution will be the same solution as the one given in Eq. (4.2). Naher et al. [22] studied the solution of the SK Eq. (4.2) using the exp-function method and they found the solution was

$$w(x,t) = a_1 + \frac{1}{1 + \cosh(x - (45a_1^2 + 15a_1 + 1)t)}. \tag{4.6}$$

If we set $a_1 = 0$ in Eq. (4.6), then

$$w(x,t) = \frac{1}{2}\sec h^2\left(\frac{1}{2}(x - t)\right)$$

which the same solution produces by Eq. (4.2) if we choose $r_1 = 1$.

Conclusions

In this paper, we study class of the fifth order nonlinear evolution equation (FNEE) with variable coefficients of t:

$$w_t + a(t)ww_{xxx} + b(t)w_x w_{xx} + c(t)w^2 w_x + w_{xxxxx} = 0 \tag{5.1}$$

where $w = w(x,t)$ and the coefficients $a(t), b(t)$ and $c(t)$ are function of the variable t. We use the simplified bilinear method based on a transformation method combined with the Hirota's bilinear sense. We derive the multiple soliton solutions for Eqs. (5.1). In addition, we derive the multiple singular soliton solutions for the problem under discussion. Also, we present some analysis of the parameters such as the soliton amplitude and the characteristic line.

We compare our results with the solutions of Choudhuri et al. [23], Mia et al. [18], Abdollahzadeh et al. [20], Ghasemi et al. [21], and Naher et al. [22] and we find that the proposed method

generate their solitons. For this reason, we consider their problems as special case of the problem under discussion. This comparison shows that

- Choudhuri et al. [23] studied only the case when $A = a(t), B = b(t)$, and $C = c(t)$ are constants. They derived the conditions $B = 2A, C = \frac{3A^2}{10}$. In addition, they use the homogeneous balance method to derive the single soliton solution which is given by

$$w(x,t) = \frac{20}{A} \frac{c_0 c_1 \sqrt{v} e^{\sqrt[4]{v}(x+vt)}}{\left(c_0 + c_1 e^{\sqrt[4]{v}(x+vt)}\right)^2}.$$

Their results are special case of our results. If we choose $c_0 = c_1 = 1$, $v = r^4$, then their single soliton solution is the same single soliton solution which is given in Eq. (4.2).

- Mia et al. [18] solved the Caudrey-Dodd-Gibbon equation and their solution was given by

$$w(x,t) = k^2 - k^2 \tanh^2(kx - 16k^5 t). \tag{5.2}$$

They found that the necessary conditions are $a(t) = b(t) = 30$ and $c(t) = \frac{a^2(t)}{5} = 180$. If we choose $k = \frac{r_1}{2}$, the solution in Eq. (4.4) becomes

$$w(x,t) = \frac{r_1^2}{4} - \frac{r_1^2}{4} \tanh^2\left(\frac{r_1}{2}x - \frac{r_1^5}{2}t\right) = \frac{r_1^2}{4} \sec h^2\left(\frac{r_1}{2}x - \frac{r_1^5}{2}t\right)$$

which the same solution given in Eq. (4.2). This means that the proposed method is also solving Caudrey-Dodd-Gibbon equation.

- Wazwaz [19] solved Eq. (4.3) and his solution was given by

$$w(x,t) = -\mu^2 \csc h^2(\mu x - 16\mu^5 t).$$

If we choose $\eta = \frac{r_1}{2}$, his solution will be the same as well as the one in Eq. (3.3).

- Abdollahzadeh et al. [20] solved Eq. (4.3) using and he found that the solution was given by

$$w(x,t) = \frac{\alpha^2(\cosh(\alpha(x - \alpha^4 t)) + \sinh(\alpha(x - \alpha^4 t)))}{(1 + \cosh(\alpha(x - \alpha^4 t)) + \sinh(\alpha(x - \alpha^4 t)))^2}.$$

If we choose $\alpha = r_1^2$, their solution will be the same solution as the one given in Eq. (4.2). Ghasemi et al. [21] used the homotopy perturbation method for solving the Sawada-Kotera (SK) Equation

$$w_t + 15ww_{xxx} + 15w_x w_{xx} + 45w^2 w_x + w_{xxxxx} = 0.$$

and they found the solution was

$$w(x,t) = 2k^2 \sec h^2(k(x - 16k^4 t - x_0)).$$

If we choose $k = \frac{r_1}{2}$ and $x_0 = 0$, then their solution will be the same solution as the one given in Eq. (4.2).

- Naher et al. [22] studied the solution of the SK Eq. (4.2) and they found the solution was

$$w(x,t) = a_1 + \frac{1}{1 + \cosh(x - (45a_1^2 + 15a_1 + 1)t)}. \tag{5.3}$$

If we set $a_1 = 0$ in Eq. (4.6), then

$$w(x,t) = \frac{1}{2} \sec h^2\left(\frac{1}{2}(x - t)\right)$$

which the same solution produces by Eq. (4.2) if we choose $r_1 = 1$.

From these comparisons, we conclude that the proposed method is efficient and the solutions produced by the simplified bilinear method based on a transformation method combined with the Hirota's bilinear sense are correct.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this article.

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.rinp.2018.01.039.

References

[1] Hereman W, Zhuang W. Macsym: A program for the Hirota method. In: 13th World congress, computation and applied mathematics, vol. 2; 1991. p. 842–63.

[2] Hereman W, Zhuang W. Symbolic computation of solitons with Macsyma. Comput Appl Math II: Differen Eqs 1992;2:287–96.

[3] Hietarina J. A search for bilinear equations passing Hirota's three-soliton condition. II. mKdV-type bilinear equations. J Math Phys 1987;28:2094–101.

[4] Awawdeh F, Jaradat HM, Al-Shara' S. Applications of a simplified bilinear method to ion-acoustic solitary waves in plasma. Eur Phys J D 2012;66:1–8.

[5] Alquran M, Jaradat HM, Al-Shara' S, Awawdeh F. A new simplified bilinear method for the n-soliton solutions for a generalized FmKdV equation with time-dependent variable coefficients. Int J Nonlinear Sci Numer Simul 2015;16:259–69.

[6] Jaradat HM, Awawdeh F, Al-Shara' S, Alquran M, Momani S. Controllable dynamical behaviors and the analysis of fractal burgers hierarchy with the full effects of inhomogeneities of media. Romanian J Phys 2015;60:324–43.

[7] Jaradat HM, Al-Shara' S, Awawdeh F, Alquran M. Variable coefficient equations of the Kadomtsev-Petviashvili hierarchy: multiple soliton solutions and singular multiple soliton solutions. Phys Scr 2012;85:1.

[8] Awawdeh F, Al-Shara' S, Jaradat HM, Alomari AK, Alshorman R. Symbolic computation on soliton solutions for variable coefficient quantum Zakharov-Kuznetsov equation in magnetized dense plasmas. Int J Nonlinear Sci Numer Simul 2014;15:35–45.

[9] Jaradat HM. New solitary wave and multiple soliton solutions for the time-space fractional boussinesq equation. Italian J Pure Appl Math 2016;36:367–76.

[10] Alsayyed O, Jaradat HM, Jaradat MMM, Mustafa Zead, Shatat Feras. Multi-soliton solutions of the BBM equation arisen in shallow water. J Nonlinear Sci Appl 2016;9(4):1807–14.

[11] Jaradat HM. Dynamic behavior of traveling wave solutions for a class for the time-space coupled fractional kdV system with time-dependent coefficients. Italian J Pure Appl Math 2016;36:945–58.

[12] Wazwaz AM. Multiple soliton solutions for the (2+1)-dimensional asymmetric Nizhnik Novikov Veselov equation. Nonlinear Anal 2010;72:1314–8.

[13] Wazwaz AM. Completely integrable coupled KdV and coupled KP systems. Commun Nonlinear Sci Numer Simul 2010;15:2828–35.

[14] Wazwaz AM. Multiple-soliton solutions for the Boussinesq equation. Appl Math Comput 2007;192:479–86.

[15] Hirota R. Direct methods in soliton theory. In: Bullough RK, Caudrey PJ, editors. Solitons. Berlin: Springer; 1980.

[16] Hirota R. Exact N-soliton solutions of a nonlinear wave equation. J Math Phys 1973;14:805–9.

[17] Yu X, Gao YT, Sun ZY, Liu Y. N-soliton solutions, Bäcklund transformation and Lax pair for a generalized variable-coefficient fifth-order Korteweg–de Vries equation. Phys Scr 2010;81:045402.

[18] Mia AS, Akter T. Some exact solutions of two fifth order KdV-type nonlinear partial differential equations. J Partial Differen Eqs 2012;25(4):357–67.

[19] Wazwaz AM. Multiple-soliton solutions for the fifth order Caudrey–Dodd–Gibbon (CDG) equation. Appl Math Comput 2008;197:719–24.

[20] Abdollahzadeh M, Hosseini M, Ghanbarpour M, Shirvani H. Exact travelling solutions for fifth order Caudrey-Dodd-Gibbon equation. Int J Appl Math Comput 2010;2(4):81–90.

[21] Ghasemi M, Fardi M, Tavassoli Kajani M, Khoshsiar Ghaziani R. Numerical solution of fifth order KdV equations by homotopy perturbation method. Math Sci 2011(5):169–81.

[22] Naher H, Abdullah FA, Akbar MA, Mohyud-Din ST. Some new solutions of the higher-order Sawada-Kotera equation via the exp-function method. Middle-East J Sci Res 2012;11(12):1659–67.

[23] Choudhuri A, Talukdar B, Dattab SB. On a generalized fifth-order integrable evolution equation and its hierarchy. Zeitschrift f ür Naturforschung A 2012;61(1–2):7–15.