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PHASE II NON-PARAMETRIC AND SEMI-PARAMETRIC PROFILE MONITORING

BY

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ABSTRACT

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The emergence of new technologies, as well as the availability of large amounts of information related to quality measurements, have shed the light to a new type of response characteristic called profile. Profiles are important when the quality of a large data process is represented by a relationship between the dependent variable and one or more independent variables. They are fitted using regression techniques and their performances are monitored by statistical process control (SPC). Most of the previous studies focused on monitoring the profiles assuming that, the model is correct with no model misspecification (parametric models). However, the parametric models may not perfectly fit the relationship of the dependent variable with the independent variable(s). Thus, this study considers profile monitoring via two non-parametric techniques and two semi-parametric techniques. The first non-parametric technique is the fitted values to the data, while the second is the fitted values to the residuals obtained from the parametric fit. Moreover, the first semi-parametric technique combines both parametric and of the non-parametric fits to the raw data (model robust regression technique 1 (MRR1), while the second one combines both parametric fit to the raw data and the non-parametric fit to the residuals obtained from the parametric fit (model robust regression technique 2 (MRR2). Also, according to the flexibility of linear mixed models (LMM), it was incorporated into different model fits. Thus, the initial portion of this research focuses on two methods for Phase II analysis, namely, MCUSUM and MEWMA statistics, to promote monitoring of the slope of LMM. Simulation study and real-data applications were carried out to compare the performances of the parametric charts with the proposed charts based on Average Run Length (ARL), standard deviation run length (SDRL) and Average Time to Signal (ATS) considering different profile sizes, sample sizes, and level of misspecifications for correlated and uncorrelated data. Furthermore, the overall abilities of the charts were evaluated by extra quadratic loss (EQL) criterion. The research demonstrates that the two proposed semi-parametric techniques had the best performances and higher sensitivities in detecting shifts.

DEDICATION

My loving family, Mother, Father, Sisters and Yousef.

My unique friends, all of them, especially Mashael, Fatma and Areej

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CHAPTER 1 : BACKGROUND AND INTRODUCTION

1.1. Introduction

The “degree of excellence” or “Quality” is what all organizations and people are looking for. It is “the degree to which a set of inherent characteristics fulfills requirements.” (ISO 9000:2005 – 3.1.1)

Every company aims to satisfy its stakeholders by corresponding to a set of certain standards; this happens by knowing the reasons behind any random variation, either due to a common cause variation or a special cause variation. The common causes are considered a part of any process; these causes are always present, predictable and can be reduced by improving the process. A process operating is said to be in-control if only common cause variations are present, whereas the process is out-of-control if special cause variations exist, which are not a part of the process neither predictable. In order to observe the occurrence of these causes, the statistical process control (SPC) method is used. It contains tools that monitor the manufacturing process by using statistical techniques to reduce variations. One of the most useful tools is the control chart that was developed by Walter A. Shewhart in the 1920s.

The quality of a large data process is represented by a relationship (linear or non-linear) between the dependent variable and independent variable(s);this relationship is known as a “profile” by referring to Kang and Ablin (2000). Profiles are fitted using regression techniques and monitoring these profiles using statistical control charts is known as “profile monitoring.” Profile monitoring fits models by regression and separates common causes from special causes variations.

There are two phases for profile monitoring using control charts: Phase I and phase II. The most important job for Phase I is to eliminate the out-of-control points from the process and to estimate the in-control parameters. While in Phase II the charts to monitor the variations of the parameters are set up, and the hidden shifts are detected based on the targeted values obtained in Phase I.

1.2. Importance of Statistical Process Control

A statistical process control (SPC) is used to monitor the quality in any production by eliminating variations from occurring. It is a primary tool for controlling a process. Some of the advantages of SPC are:

- Lower costs of quality
- Increase of process productivity
- A better understanding of the process
- Reliable data for future improvements in the process

1.3. Definition of Terms Related to Profile Monitoring

1.3.1 Control chart

The SPC tool is used to identify whether the process is in-control or not. It is used to understand the process performance through a graphical representation of the whole process. The objective of control charts is to determine the special cause variations to prevent them from occurring again. Furthermore, important process parameters are estimated through control charts to determine the process capability. The graph contains a centerline (drawn corresponding to the average value), upper control limits and lower control limits (computed from process data). Control charts depend on the parameters estimated by the model from consecutive profile data observed over time, the process is out-of-control if a point is falling outside any of these limits. Moreover, if the points follow a systematic pattern or form a trend yet and are falling inside the boundaries. This is an indication that the process is out-of-control and action should be taken.

There are two types of control charts:

- *Univariate control chart*: a graphical representation for only one quality characteristic, for example, Shewhart, cumulative sum (CUSUM), exponentially weighted moving average (EWMA), $\bar{X}$ and R $charts$.
- *Multivariate control chart*: is a graphical representation for more than one quality characteristic, for example, Hotelling's T^2, multivariate exponentially weighted moving average (MEWMA) and multivariate cumulative sum (MCUSUM).

This study focuses on MCUSUM and MEWMA control charts to be used in Phase II; because of their high ability in detecting process shifts quickly.

- *Cumulative Sum (CUSUM) control charts*

They were announced in 1954 by Page and used to detect small shifts in the mean (or variance). The main feature of CUSUM is that consecutive values of a variable are compared with the target value computed in Phase I, and the cumulative sum of deviations from this value is plotted. If any of the cumulating values exceeds the control limits computed in Phase I, then this is an indication that a change in the mean (or variance) level of the variable has occurred and the process is considered to be out-of-control. Then, the process should be stopped and the causes behind this change are inspected. CUSUM charts are univariate charts and are extended to MCUSUM charts which are multivariate charts and were introduced by Crosier in 1988 (their formulas are given in chapter 4).

- *Exponentially Weighted Moving Average (EWMA) control charts*

They were introduced by Roberts in 1959, it is used to monitor small shifts in the process mean (or variance). EWMA is a statistic used to find the average of data where less weight is given to the oldest observations compared to most recent observations. Hence, the data should be time-ordered. One of the uses of the EWMA chart is forecasting the next period observations; this helps analysts in taking different actions to avoid the occurrence of an out-of-control state. Moreover, when the data is not normally distributed or auto-correlated, the EWMA chart performs well. EWMA charts are univariate charts and are extended to MEWMA charts, which are multivariate charts and were introduced by Lowry et al. in 1992 (their formulas are given in chapter 4).

1.3.2. Phase I Analysis

It analyzes a set of historical process data to construct trial control limits for determining the state of the process over time. It is used to determine the out-of-control profiles to be eliminated from the process. Moreover, it estimates profile in-control parameters (such as mean and variance) to be used in Phase II. The

Probability of Signal (POS) is the method used to check the performance of control charts used in this Phase, and it is the probability of getting at least one point outside any of the limits.

1.3.3. Phase II Analysis

The success of Phase II analysis depends on the successful analysis done in phase I; since in this phase the trial control limits constructed in Phase I are used to defining if the process is in-control using new samples of data. This phase is an "online" process for monitoring estimated parameters over time and aims to detect if a special cause variation occurs as soon as possible. The Average Run Length (ARL) is one of the criteria that is used to compare the performances between different control chart methods. It is the mean of samples required until the first out-of-control signals (when a change in the process parameters occurs).

ARL_0: the mean of the samples until a control chart signals given that the process is in-control.

ARL_1: the mean of samples until a control chart signals given that the process is out-of-control.

Thus, ARL_0 must be as large as possible as it is an indication that the process is on target, while ARL_1 must be as small as possible for a quick detection for any shift in the process parameters). Furthermore, Average Time to Signal (ATS) is another criterion used to measure the performance of any control chart, it is the number of time periods (h) that occur until a signal is generated and it is given as: ATS = (ARL) * h. Moreover, Standard Deviation Run Length is the standard deviation of the samples until a control chart signals.

Montgomery (2005), Mahmoud and Woodall (2004) provide more details regarding Phase I and Phase II.

1.3.4. Profile Monitoring

It is a quality control technique used when the process data follow a profile at each time period. It fits different models by using regression and separates the common cause from special cause variations (a combination of model fitting and SPC techniques). Woodall et al. (2004) gave a good introduction and examples of its application.

1.3.5. Profile Monitoring Examples

Profile monitoring was used by Mahmoud and Woodall (2004) in linear calibration, where different concentrations were collected by analytical chemistry procedures. The curves were fitted by linear models to guarantee that the measurement equipment is calibrated. Non-linear profiles were considered by Williams et al. (2003), where particleboard density was measured at equally spaced locations and the result was higher density at the edges compared to the density in the middle. Ding et al. (2006) used wavelet transforms (which are more complicated compared to linear on non-linear models) to approximate the profile of the forging cycle of a stamping process, the wavelet coefficients were monitored to determine whether the forging cycles differ from each other or not.

1.4. Penalized Spline Estimation

Regression analysis is an important part in statistics that is used to explain the relationship between the random variables in a population (or sample). Parametric regression usually has some assumptions regarding the data (errors are normally distributed, or a linear relationship exists). However, practically, these assumptions are not realistic, as the underlying distribution of the data may not always be known or may not be assumed correctly, which may result in biased estimations. Therefore, parametric regression may be misleading as they are based on false assumptions. Non-parametric regression is an alternative approach used due to its flexibility, as it eliminates the assumptions of the parametric approach. There are different methods available in non-parametric regression used to fit the model between the dependent variable and the independent variables. In this study, we will focus on the penalized spline (p-spline) method for fitting two models, the first model is the raw data fit while the second is for the residuals obtained by the parametric approach. A spline is a continuous function that is used to generate a smooth curve to pass through a set of points. Usually, the n-degree polynomials are generated; thus, n-degree spline functions are required to join these polynomials; the polynomials are tied together by knots to produce a smooth curve joining them. The number of knots and their locations must be chosen accurately in order to avoid over or underfitting the data, but as there is no optimal solution exists to the number of knots or their locations; penalization is used to put weights on the splines to avoid overfitting and at the same time to allow the accurate fit to the data.

The p-spline smoothing is represented by:

$$y_i \sim N(m(x_i), \sigma_\in^2) \quad i = 1, \dots, n \tag{1.1}$$

where $m(x) = \beta_0 + \beta_1 x + \dots + \beta_p x^p + \sum_{j=1}^{K} u_j (x - k_j)_+^p$ is an unspecified smooth function. $\beta_0 + \beta_1 x + \dots + \beta_p x^p$ represents a parametric polynomial model and $\sum_{j=1}^{K} u_j (x - k_j)_+^p$ represents a polynomial basis function. $k_1, \dots, k_K$ are the K knots. The model in (1.1) can be re-written considering the given $m(x)$:

$$y_i \sim N(X\beta + Zu, \sigma_\in^2 I_n) \quad i = 1, \dots, n \tag{1.2}$$

The fitted model of (1.2) can be fitted by using the parametric ordinary least squares (OLS) method to get:

$$\hat{y} = C(C'C)^{-1}C'y \tag{1.3}$$

where $C = [X, Z], X = \left[1, x_i, \dots, x_i^p\right]$ and $Z = \left[(x_i - k_1)_+^p, \dots, (x_i - k_K)_+^p\right]$.

1.5. Model Robust Regression

Parametric and non-parametric estimations have some deficiencies; as mentioned before, biased estimations and model misspecifications are the consequences of using parametric approaches, while for the non-parametric approaches, the fits are subjected to high variance as the estimates depend on the data itself. Therefore, an alternative methodology is the use of a semi-parametric method that combines both parametric and non-parametric methods. Einsporn and Birch (1993) proposed a semi-parametric method, model robust regression 1 (MRR1) for modeling the mean response. Their technique combined both parametric and non-parametric fits to the raw data by using the mixing parameter: $\lambda \in [0,1]$. The estimated mean responses by MRR1 are obtained as:

$$\hat{y}^{MRR1} = (1-\lambda)\hat{y}^{OLS} + \lambda\hat{y}^{LLR}$$

where $\hat{y}^{OLS}$ is the ordinary least square (OLS- parametric fit) and $\hat{y}^{LLR}$ is the local linear regression (LLR (non-parametric fit)) vectors containing the mean estimates. Another semi-parametric technique is model robust regression 2 (MRR2) that was proposed by Mays et al. (2000), MRR2 takes the advantages of parametric and non-parametric techniques and avoids their disadvantages by combining the parametric fit (OLS) to the raw data with the non-parametric fit (LLR) to the residuals obtained from the parametric fit by using the mixing parameter: $\lambda \in [0,1]$. The estimated mean responses by MRR2 are obtained as:

$$\hat{y}^{MRR2} = \hat{y}^{OLS} + \lambda\hat{r}^{LLR}$$

where $\hat{r}^{LLR}$ contains the fitted residuals r from the parametric fit $\hat{y}^{OLS}$.

1.6. Motivation

Because of the advances in different technologies nowadays and the availability of large amounts of information related to quality measurements, SPC to monitor the quality of data-rich processes became important. As mentioned above, the profile is represented by a relationship between the dependent variable and one or more independent variables. Most of the previous studies focused on monitoring the profiles based on assuming that the model is correct with no misspecification (parametric model). However, these models may not perfectly fit the relationship between the explanatory variable and the response(s); resulting in problems caused by model misspecification, such as biased estimates. Lately, non-parametric techniques are investigated to address the model misspecification problem, as it makes no assumptions about the relationship between variables. These techniques are flexible but will give high variances are subjected to their fits and they might not provide a good estimation if a small number of data is available in each profile. Therefore, semi-parametric (MRR1 and MRR2) techniques mentioned above will be used in this study to fit the models according to their accurate estimations compared to parametric and non-parametric estimations.

Another type of misspecification is using a fixed model that ignores the correlation within profiles and assumes all profiles are independent of each other, which is an unrealistic assumption. Jensen et al. (2008) proposed the use of LMM on Phase I, where the correlation structure within a profile was taken into account. They found that their approach is preferred compared to the other approach that ignores the correlation structure.

Hence, based on the flexibility of LMM, this study adopts two non-parametric and the two semi-parametric techniques for performing Linear Mixed Models (LMM) profile data in Phase II analysis by introducing new MEWMA & MCUSUM control charts. The proposed MEWMA and MCUSUM control charts will help in detecting certain types of changes due to misspecified, out-of-control model and taking into consideration the correlation within and among profiles. The Average Run Length (ARL) criterion (and Average Time to signal (ATS)) will be utilized for performance comparison of the proposed control charts with the state-of-arts. The comparison will be performed based on a comprehensive simulation study and three real-data applications.

CHAPTER 2: LITERATURE REVIEW

Many studies were conducted on profile monitoring, considering different regression techniques (parametric, non-parametric and semi-parametric). Also, considering various types of models (linear, non-linear, mixed models, etc.) for control charting in Phase I and Phase II.

Kang and Albin (2000) introduced two approaches to monitor linear profiles for fixed effects in Phase II, the first was monitoring slope and intercept parameters by Hotelling's T^2 control chart, while the second was monitoring average residuals between sample profiles and reference profile followed by EWMA chart and R chart. Kim et al. (2003) suggested two-control charts for monitoring a process characterized by a linear profile at phase II. It is similar to the method proposed by Kang and Albin, where instead of using statistics based on the successive samples of deviations from the in-control line, the independent variable was coded to zero average value and the estimated regression coefficients (Y-intercept and slope) from each sample to construct two univariate EWMA charts. In addition, an EWMA chart was used for monitoring any increase in the error variance. Average run length (ARL) was used to compare the performance of each approach, concluding that the $EWMA_3$ approach performed better compared to the EWMA/R approach in detecting sustained shifts in any of the regression coefficients or in increases in the error variance.

Mahmoud and Woodall (2004) proposed profile monitoring to monitor the process variance of multiple regression using indicator variables in phase I, where they suggested using F test in combination with a univariate T^2 chart; the process is out-of-control if the equality of parameters is rejected. They compared their proposed method with an existing T^2 chart and they recommended developing a robust profile monitoring method.

Jensen et al. (2008) used linear mixed models (LMM) in monitoring linear profiles at phase I. Correlation within a profile was taken into account and simulation was performed for both balanced and unbalanced data. For balanced data, AR(1) model was used as the correlations were assumed to be equal between successive observations, while for unbalanced data, an exponential model was used as the data are unequally spaced. Their proposed T^2 statistic depended on the sample mean and the successive-differences variance-covariance matrix. It was compared with the results of the least square (LS) approach (random effects and correlation were ignored), concluding that the proposed LMM is better than LS approach with unbalanced or missing data scenarios.

Jensen and Birch (2009) conducted a study on profile monitoring using non-linear mixed models (NLMM) in phase I, where the correlation structure within profiles was considered. They found that the efficiency of NLMM was higher compared to the efficiency of the non-linear model (NLM) where random effects are ignored.

Abdel-Salam (2009) proposed a semi-parametric profile monitoring (MMRPM) study on linear mixed models (MMRPM) using MRR1 regression method. He evaluated both of the correlated and uncorrelated profile datasets for different model misspecification levels, shift location, observations number per profile and in-control and out-of-control situations. The MMRPM method was compared with parametric and non-parametric methods and based on the results of the probability of signals

(POS) and the simulated integrated mean square error (SIMSE); the MMRPM method had the best performance.

Saghaei et al. (2009) proposed a CUSUM method to monitor simple linear profiles (parameters: intercept, slope and error variance) at Phase II. This method used three distinct univariate control charts ($CUSUM_3$) for each parameter, as it is easier to detect any step shifts. Their study showed that the proposed CUSUM had the best performance in observing medium to large shifts for the intercept parameter when compared to previous methods such as EWMA/R, $EWMA_3$, MEWMA, T^2 and MCUSUM/χ^2. It also had a superior performance in observing large shifts occurring in the slope parameter. Furthermore, their proposed CUSUM performed better than other methods under positive standard deviation shifts. Larger sample size led to a better performance of detecting all types of shifts.

Noorossana et al. (2010) proposed three control charts (single MEWMA chart, combined MEWMA & chi-square chart (MEWMA/$\mathcal{X}^2$) and three MEWMA charts ($MEWMA_3$)) for monitoring intercept, slope and variance-covariance matrix of multivariate simple linear profiles in phase II. Average run length criterion was used to compare the performances of the proposed charts, for detecting shifts in intercept, MEWMA/$\mathcal{X}^2$ performed better than the other charts, for detecting shifts in slope, both MEWMA and MEWMA/$\mathcal{X}^2$ had superior performances compared to $MEWMA_3$ and for monitoring the variance-covariance matrix, MEWMA/$\mathcal{X}^2$ and $MEWMA_3$ performed better than MEWMA chart.

Qiu et al. (2010) monitored non-parametric mixed models in phase II where they have taken the correlation within profiles in consideration, they incorporated local linear kernel smoothing with exponentially weighted moving average (EWMA) chart and they only considered the shift in the fixed effects term, by simulation, they showed the effectiveness of their proposed method.

Abdel-Salam et al. (2012) proposed a semi-parametric (mixed model robust profile monitoring (MMRPM) approach to Phase I profile monitoring. Their proposed method is a combination of parametric fit and p-spline as a non-parametric fit. Their simulation showed that the MMRPM method is robust to all levels of model misspecification when compared to parametric and non-parametric fits. Moreover, the simulated integrated mean square (SIMSE) of the out-of-control case is estimated with uncorrelated error structure and it was found that as the observation number per profile increases, the SIMSE decreases, which means the power of the MMRPM method in detecting shifts increases. Furthermore, they found that as the correlation increases, the performance of MMRPM increases for all values of sample size and profile.

Narvand et al. (2013) proposed a LMM approach that accounts for correlation within linear profiles at phase II. Three multivariate control charts: MCUSUM, MEWMA and T^2 were used to detect shifts in any of the estimated parameters (intercept, slope and standard deviation). Average run length (ARL) criterion was used to compare the performances of these methods, MCUSUM and MEWMA methods performed better than the Hotelling T^2 control chart in detecting intercept and slope parameters shifts of linear profile. Moreover, the Hotelling T^2 control chart had better performance in detecting the standard deviation shift compared to MEWMA and MCUSUM methods.

Chou et al. (2013) focused on developing a proper process strategy for monitoring multiple correlated non-linear profiles. They studied two methods; the first method fits profiles by B-splines while the second method is similar to the first one except that the number of segments of each type of profile is larger than 1. MEWMA control chart was then used in monitoring the process and according to the on (ARL) criterion, they have found that method I perform better compared to method II when the shape of a profile changes entirely, while method II has lower ARL_1 than method I in the cases where profiles shift partially.

Noorossana et al. (2015) proposed monitoring simple linear profiles for intercept, slope and standard deviation shifts for Phase II when the explanatory variable is random; they used T^2, EWMA/R and $EWMA_3$ control charts for comparing their performances for two cases: when the explanatory variable is random and when it is fixed, they have found using average run length (ARL) that neither T^2 nor EWMA/R control charts were affected by the assumption of the random explanatory variable. Thus, they have improved the $EWMA_3$ chart where is wad found to be effective for monitoring simple linear profiles in Phase II for a random explanatory variable.

Mahmoood et al. (2018) proposed an efficient way for monitoring the parameters (intercept, slope and error variance) of linear profiles in Phase I. They have applied different sampling techniques such as ranked set sampling (RSS), double RSS (DRSS), median RSS (MRSS), double MRSS (DMRSS), extreme RSS (ERSS) and double ERSS (DERSS). They have compared the performances of their proposed $shewhart_{[R]}$-3 with an existing chart using probability to signal (PTS) and found that the proposed DRSS and DMRSS had the best performances under intercept and slope shifts, furthermore, their proposed DERSS performed better than other charts under error variance shifts.

Abbas et al. (2019) proposed new Phase II monitoring methods for linear profiles considering the random effect model. They have investigated the randomness present in the explanatory variable by Ranked set sampling (RSS) techniques. They considered the perfect correlation case between the main variables and the auxiliary variables. Moreover, they have used three distinct EWMA charts for monitoring each parameter (intercept, slope and errors variance). By comparing the ARLs of their proposed $EWMA_{x[R]}$-3 method with the existing $EWMA_{x[SRS]}$-3 method, the results showed a better performance of their proposed method in detecting step shifts in the intercept and the slope parameters, while $EWMA_{x[DRESS]}$-3 showed the best performance in detecting the errors variance shift. They concluded that RSS schemes improve the control charts capability in detecting shifts of linear profiles model.

Siddiqui and Abdel-Salam (2019) performed a study of a semi-parametric method for monitoring residuals from the parametric linear mixed-effects profiles in Phase I using Hoteling's T^2 statistic. The models were fitted for the profile average model and the cluster specified model considering both correlated and uncorrelated error scenarios. Moreover, different misspecification levels were assumed. They compared the performances of the parametric, non-parametric on raw data, non-parametric on residuals (penalized spline) and the semi-parametric (MMRPM) methods with the proposed semi-parametric (MMRRPM) method by using SIMSE, and the result of SIMSE for monitoring the residual profiles of her proposed method is less than other the methods. Furthermore, lower model misspecification SIMSE results for parametric estimation and MMRRPM are similar, while higher model misspecification led the proposed estimation to be identical to the non-parametric estimation. They concluded that MMRRPM is the best approach to monitor the LM effect profiles as it uses residuals fit from the parametric approach and non-parametric estimation.

CHAPTER 3: LINEAR MIXED MODELS (LMM)

3.1. Model Formulation

It is a type of regression model that considers both random and fixed effects. Random-effects are the variations not explained by the independent variables, while fixed effects are the variations explained by the independent variables. It is a very flexible model; it allows the errors to be independent or correlated, it is used in many different fields of data, particularly used when there is a dependency in the data, which means; it accounts the correlations within profiles. All profiles are considered a random sample from a common population distribution, but each has its own special characteristics (represented as random-effect in the model), this consideration is more realistic than the consideration of fixed-effect model; which assumes all profiles are similar and can be grouped. Verbeke and Molenberghs (2000) gave good introduction to LMM.

The general LMM form for *m* profiles is given by:

$$y_i = X_i\beta + Z_i b_i + \epsilon_i \qquad i = 1,2,\dots,m \tag{3.1}$$

where $y_{i(n_i * 1)}$ is the vector of responses for the i^{th} profile. $X_{i(n_i * p)}$ is the independent variables matrix, $\beta(p * 1)$ is the fixed effect vector associated to X_i and it's the same for all profiles. $Z_{i(n_i * q)}$ corresponds to the predictor variables with random effects vector $b_{i(q * 1)} \sim MN\,(0, D)$ for the i^{th} cluster, where $D(q * q)$ is a diagonal positive definite matrix; which means, the random effects are uncorrelated. $\epsilon_i \sim MN(0, R_i)$ is the error terms vector, the errors can be either independent or correlated and thus the model is considered flexible. Independent error means $R_{i(n_i * n_i)} = \sigma^2 I$ (I: identity matrix), while correlated errors assumes R_i to be a simple form such as autoregressive (AR) or compound symmetry (CS). Moreover, $cov(\varepsilon_i, b_i)=0$ (uncorrelated random errors ε_i and random effects b_i); resulting in the conditional model given by:

$$y_i|b_i \sim MN(X_i\beta + Z_ib_i, R_i) \tag{3.2}$$

It is assumed that Z_i is either a subset of or equal matrix to X_i, so the Z_i columns are contained in the X_i matrix with $p \geq q$. When $Z_i = X_i$. It was referred to this model as a random coefficients model by Demidenko (2004); as every fixed effect has a corresponding random effect.

The corresponding marginal model is given by:

$$y_i \sim MN(X_i\hat{\beta}, V_i) \qquad i=1,2,\ldots,m \tag{3.3}$$

where $Vi=Z_iD_iZ'_i + R_i$ is a $(n_i * n_i)$ positive definite matrix.

The correlations within a profile in model (3.1) are given by two levels, the first level of correlation results from the random effects where all measurements within a profile are correlated and the errors are uncorrelated, this reduces the model in (3.3) to $y_i \sim MN(X_i\beta, V_i)$ where $V_i=Z_iD_iZ'_i + \sigma^2 I$, while the second level of correlation results from the within profile variance-covariance matrix. A linear mixed model that doesn't use any of the correlation levels reduces the model in (3.3) to the general linear model: $y_i = X_i\beta_i + \epsilon_i$; because $Z_i = 0$ and $\epsilon_i \sim MN(0, \sigma^2 I)$.

3.2. Matrix Form

The matrix form (stacked form) of the model in (3.1) is written by:

$$Y = X\beta + Zb + \varepsilon \tag{3.4}$$

where $Y_{(N*1)}$ contains the responses for all the profiles ($N=\sum n_i$). $X_{(N*p)}$ matrix contains the X_i's and $\beta_{(p*1)}$ is the fixed effects vector. $Z_{(N*mq)}$ is the block diagonal matrix with $Z=diag(Z_i)$ and $b_{(mq*1)} \sim MN(0, B)$ represents the random effects vector with $B=diag(D)$. The vector of error terms $\varepsilon_{(N*1)} \sim MN(0, R)$ with $R=diag(R_i)$ and $cov(\varepsilon, b)=0$. The marginal model in (3.3) expressed by a matrix is $y \sim MN(X\beta, V)$, where $V_{(N*N)}=ZBZ' + R=diag(V_i)$ is a positive definite matrix. The conditional model as a matrix form is $y|b \sim MN(X\beta + Zb, R)$.

3.3. Estimation

Regression uses one or more regressors ($X_1, X_2, \dots, X_k$) in explaining the response variable y. Most regression models are of the form:

$$y=f(X_1, X_2, \dots, X_k)+\varepsilon \tag{3.5}$$

where the assumption of the error term is usually $\varepsilon \sim N(0, \sigma^2)$.

Parametric techniques such as ordinary least square (OLS) and generalized least square (GLS) are used for fitting the model assuming f is of a known form, but if the model is misspecified, this technique will be misleading; causing biased parameters estimation. Non-Parametric techniques such as Penalized-spline and local linear regression are alternative approaches used when function f is unknown; these techniques depend on the data only without knowing the distribution of it. Thus, non-parametric techniques are used in identifying mean structure when no structure exists and their fits may be more variable compared to parametric fits. However, these techniques often result in estimates with a problematic variation. Semi-parametric techniques are used to solve the problems of the previous approaches by combining parametric and non-parametric techniques to improve the quality of mean and variance estimations.

In this study, MRR1 and MRR2 are the robust methods used for fitting the model by combining the parametric fit to the raw data by using GLS with the non-parametric fit, firstly to the raw data using MRR1 and secondly to the residuals obtained from the parametric fit based on the MRR2 by using P-spline to obtain semi-parametric methods for monitoring profiles. The estimation techniques used in this study are described below:

3.3.1. Parametric Estimation

According to the distributional assumptions of the marginal model in (3.3), the fixed effects parameter estimator using GLS is given by:

$$\hat{\beta}_p = (\sum_{i=1}^{m} X'_i V_i^{-1} X_i)^{-1} (\sum_{i=1}^{m} X'_i V_i^{-1} y_i) \tag{3.6}$$

which represents the population average of all m profiles. The estimates of random deviations from population average are given by:

$$\hat{b}_{i,p} = D Z_i' V_i^{-1} \left(y_i - X_i \hat{\beta}_p\right) \tag{3.7}$$

which are the best linear unbiased predictors (“blups”). The mean and variance of $\hat{b}_{i,p}$ are given by, respectively, $\bar{\hat{\mathrm{b}}}_P = \frac{\sum_{i=1}^{m} \hat{b}_{i,P}}{m}$ and $\hat{\Sigma}_P = \frac{\sum_{i=1}^{m-1} (\hat{b}_{i+1,P} - \hat{b}_{i,P})(\hat{b}_{i+1,P} - \hat{b}_{i,P})'}{2(m-1)}$.

If LMM stacked form is used and the model is correctly specified, then the fixed-parameter estimators (for all profiles) and the estimated random effects (for each profile) vectors are consequently given by:

$$\hat{\beta}_p = (X'V^{-1}X)^{-1}(X'V^{-1}y) \quad (3.8)$$

$$\hat{b}_p = BZ'V^{-1}(y - X\hat{\beta}_p) \quad (3.9)$$

The maximum likelihood estimation (MLE) method or the restricted maximum likelihood estimation (RMLE) method can be used to estimate V (consequently, B and R are estimated). RMLE is preferred as it produces less bias estimators compared to (MLE) Schabenberger and Pierce (2002).

Thus using (3.6) and (3.7) the estimated parameter vector for the ith profile is given by:

$$\hat{\beta}_{i,p} = \hat{\beta}_p + \hat{b}^*_{i,\mathrm{p}} \quad (3.10)$$

where $\hat{b}^*_{i,\mathrm{p}}$ contains the elements of $\hat{b}_{i,p}$. The mean and variance of $\hat{\beta}_{i,p}$ are given by, respectively, $\bar{\bar{\beta}}_P = \frac{\sum_{i=1}^{m} \hat{\beta}_{i,P}}{m}$ and $\hat{\Sigma}_P = \frac{\sum_{i=1}^{m-1} (\hat{\beta}_{i+1,P} - \hat{\beta}_{i,P})(\hat{\beta}_{i+1,P} - \hat{\beta}_{i,P})'}{2(m-1)}$.

The Profile Average (PA) fit for all profiles is given by:

$$\hat{Y}^P_{PA} = X\hat{\beta}_{i,p} \quad (3.11)$$

The cluster-specific (CS) fit for the ith profile is given by:

$$\hat{Y}^P_{CS,i} = X\hat{\beta}_{i,p} + Z_i\hat{b}_{i,p} \quad (3.12)$$

Residuals are frequently used to evaluate the validity of assumptions of statistical models and help in model selection. In fixed-effect models, the residuals depend on the fixed components and on the error that is supposed to predict (single source of variability); thus they are called “pure” errors. In LMM, residuals accommodate extra source of variability and are called "confounded" residuals. The three types of residuals by referring to Hilden-Minton (1995) and Verbeke & Lesaffre (1997) are:

i) Marginal residuals:

$$\xi_i = y_i - \mathrm{E}[Y_i] = y_i - X_i\beta = Z_ib_i + \epsilon_i \tag{3.13}$$

ii) Conditional residuals:

$$\epsilon_i = y_i - \mathrm{E}[Y_i|b_i] = y_i - X_i\beta - Z_ib_i \tag{3.14}$$

iii) Best Linear Unbiased Predictors (BLUP):

$$Z_ib_i = \mathrm{E}[Y_i|b_i] - \mathrm{E}[Y_i] = \widehat{D}Z_i'\widehat{V}_i^{-1}\left(\hat{y}_i - X_i\hat{\beta}\right) \tag{3.15}$$

Hence, from (3.1) model, the residuals for PA are the marginal residuals and the residuals for CS model are the BLUPs.

3.3.2. Non-Parametric Estimation

Siddiqui and Abdelsalam (2019) followed Abdelsalam et al. (2012) P-spline's technique to model residuals from the estimated PA and CS, as among all non-parametric techniques, P-spline is well known for its flexibility to fit curves without choosing in advance a rigid form for the underlying function and penalties are attached to the number of knots included. Based on their approaches, y_{ij} is the j^{th} observation on the i^{th} profile (actual data) modeled as:

$$y_{ij} = f\left(x_{ij}\right)+, \xi_i\left(x_{ij}\right) + \varepsilon_{ij} \qquad i = 1,2,\dots,m \quad j = 1,2,\dots,n \tag{3.16}$$

where $f(x_{ij})$ is the true mean response function for all profiles (PA), $\xi_i(x_{ij})$ represents the random difference between i^th CS curve and the PA curve, it is a smoother function. ε_{ij} is the random error associated with y_{ij} . Siddiqui and Abdelsalam replaced $f(x_{ij})$ & $\xi_i(x_{ij})$ by $f(r_{ij})$ & $\xi_i(r_{ij})$ where r= $y - \hat{y}^{p}_{MIX}$ represents the residuals from the estimated parametric LMM and found them by using the approximate of the second-order polynomial base function.

$$f(x_{ij}) \approx \beta_0 + \sum_{l=1}^{p} \beta_i x_{ij}^{l} + \sum_{k=1}^{k_1} u_k \left|x_{ij} - K_k\right|^{p} \quad i = 1,2,\dots,m \;\; j = 1,2,\dots,n_i \quad (3.17)$$

$$f(r_{ij}) \approx \beta_0 + \sum_{l=1}^{p} \beta_i r_{ij}^{l} + \sum_{k=1}^{k_1} u_k \left|r_{ij} - K_k\right|^{p} \quad i = 1,2,\dots,m \;\; j = 1,2,\dots,n_i \quad (3.18)$$

where p is the order of polynomial basis, k_1 is the number of knots used in the non-parametric portion of PA curve and K_1, …, K_{k1} are the locations of the knots. $\sum_{l=1}^{p} \beta_i x_{ij}^{l}$ and $\sum_{l=1}^{p} \beta_i r_{ij}^{l}$ are the parametric components. While $\sum_{k=1}^{k_1} u_k \left|x_{ij} - K_k\right|^{p}$ and $\sum_{k=1}^{k_1} u_k \left|r_{ij} - r_k\right|^{p}$ are the non-parametric (p-spline) components.

$$\xi_i(x_{ij}) \approx b_{i0} + \sum_{l=1}^{p} b_{ij} x_{ij}^{l} + \sum_{k=1}^{k_2} t_{ipk} \left|x_{ij} - K_k\right|^{p} \; i = 1,2,\dots,m \; j = 1,2,\dots,n_i \quad (3.19)$$

$$\xi_i(r_{ij}) \approx b_{i0} + \sum_{l=1}^{p} b_{ij} r_{ij}^{l} + \sum_{k=1}^{k_2} t_{ipk} \left|r_{ij} - K_k\right|^{p} \quad i = 1,2,\dots,m \;\; j = 1,2,\dots,n_i \quad (3.20)$$

$\xi_i(x_{ij})$ & $\xi_i(r_{ij})$ are the p-spline basis for CS curve and p represents the order of polynomial basis with the intercept and the parameters of random effects. The parametric components are $b_{i0} + \sum_{l=1}^{p} b_{ij} x_{ij}^{l}$ and $b_{i0} + \sum_{l=1}^{p} b_{ij} r_{ij}^{l}$ while the non-parametric (p-spline) components are $\sum_{k=1}^{k_2} t_{ipk} \left|x_{ij} - K_k\right|^{p}$ and $\sum_{k=1}^{k_2} t_{ipk} \left|r_{ij} - K_k\right|^{p}$, where k_2 is the number of knots used.

The estimated PA curve and CS curve for the ith profile using p-spline regression are given by:

$$\hat{y}_{PA}^{NP} = X_i\hat{\beta} + Z_i\hat{u} \qquad i = 1,2,\dots,m \tag{3.21}$$

$$\hat{y}_{CS,i}^{NP} = X_i\hat{\beta} + Z_i\hat{u} + X_i\hat{b}_i + E_i\hat{t}_i \qquad i = 1,2,\dots,m \tag{3.22}$$

where $\hat{\beta}$, $\hat{u}$, $\hat{b}_i$ and $\hat{t}_i$ are the estimators of the coefficients for non-penalized polynomial basis, radial basis and the knots sequence, respectively. More details regarding this technique and its assumptions are found in Abdelsalam's, Birch's & Jensen's and Siddiqui's researches. Note that, the estimated PA curve and CS curve for the i^{th} profile using residuals p-spline regression have the same components as (3.21) and (3.22) respectively, but instead of fitting the data, the residuals obtained parametrically are fitted with $\hat{y}_{PA}^{NPR}$ and $\hat{y}_{CS,i}^{NPR}$ notations. Furthermore, define the estimated random effects vector from the non-parametric estimation to be: $\hat{\gamma}_{i,NP} = [\hat{b}_i\ \hat{t}_i]'$, where $\hat{b}_i$ is the estimated random effects for the spline component vector and $\hat{t}_i$ is the knot location for the i^{th} profile vector, where $(b_{i0}, b_{i1}, \dots b_{ip})' \sim MN(0, \Sigma_b)$ and $t_{ipk2} \sim MN(0, \sigma_t^2)$. The mean and variance of $\hat{\gamma}_{i,NP}$ are given by,respectively, $\bar{\gamma}_{NP} = \frac{\sum_{i=1}^{m}\hat{\gamma}_{i,NP}}{m}$ and $\hat{\Sigma}_{NP} = \frac{\sum_{i=1}^{m-1}(\hat{\gamma}_{i+1,NP} - \hat{\gamma}_{i,NP})(\hat{\gamma}_{i+1,NP} - \hat{\gamma}_{i,NP})'}{2(m-1)}$.

Note that, the estimated random effects using residuals p-spline regression have the same components as $\hat{\gamma}_{i,NP}$ but instead of fitting the data, the residuals obtained parametrically are fitted with $\hat{\gamma}_{i,NPR}$ notation, with mean and variance given by, respectively, $\bar{\gamma}_{NPR} = \frac{\sum_{i=1}^{m}\hat{\gamma}_{i,NPR}}{m}$ and $\hat{\Sigma}_{NPR} = \frac{\sum_{i=1}^{m-1}(\hat{\gamma}_{i+1,NPR} - \hat{\gamma}_{i,NPR})(\hat{\gamma}_{i+1,NPR} - \hat{\gamma}_{i,NPR})'}{2(m-1)}$.

3.3.3. Semi-Parametric Estimation

Based on the proposed Mixed Model Robust Profile Monitoring (MMRPM) method by Abdelsalam, Birch and Jensen (2012), the MMRPM fit for the PA and CS for the ith profile are given by:

$$\hat{y}_{PA}^{MMRPM} = (1 - \hat{\lambda}_{PA,MMRPM})\hat{y}_{PA}^{P} + \hat{\lambda}_{PA,MMRPM}\hat{y}_{PA}^{NP} \qquad (3.23)$$

$$\hat{y}_{CS,i}^{MMRPM} = (1 - \hat{\lambda}_{CS,MMRPM})\hat{y}_{CS,i}^{P} + \hat{\lambda}_{CS,MMRPM}\hat{y}_{CS,i}^{NP} \qquad (3.24)$$

Since MMRPM is a combination of parametric and non-parametric fits to the data, the estimated random effects vector is given by:

$$\hat{\psi}_{i,MMRPM} = \begin{bmatrix} (1-\hat{\lambda}_1)\hat{b}_{i,P} \\ \hat{\lambda}_1\hat{\gamma}_{i,NP} \end{bmatrix}' \qquad (3.25)$$

where $\hat{\lambda}_1$ is given in equation 3.30. Furthermore, both mean and variance are given by, respectively, $\bar{\psi}_{MMRPM} = \frac{\sum_{i=1}^{m}\hat{\psi}_{i,MMRPM}}{m}$ and $\hat{\Sigma}_{MMRPM} = \frac{\sum_{i=1}^{m-1}(\hat{\psi}_{i+1,MMRPM}-\hat{\psi}_{i,MMRPM})(\hat{\psi}_{i+1,MMRPM}-\hat{\psi}_{i,MMRPM})'}{2(m-1)}$.

According to the proposed Mixed Model Robust Residuals Profile Monitoring (MMRRPM) method by Siddiqui and Abdelsalam, the MMRRPM fit for the PA and CS for the ith profile are given by:

$$\hat{y}_{PA}^{MMRRPM} = \hat{y}_{PA}^{P} + \hat{\lambda}_{PA,MMRRPM}\hat{r} \qquad (3.26)$$

$$\hat{y}_{CS,i}^{MMRRPM} = \hat{y}_{CS,i}^{P} + \hat{\lambda}_{CS,MMRRPM}\hat{r} \qquad (3.27)$$

where $r = y - \hat{y}_{PA}^{P}$ and $\hat{r} = \hat{r}^{NP}$.

Since MMRRPM is a combination of parametric fit to the data and non-parametric fits to the residuals obtained parametrically, the estimated random effects vector is given by:

$$\hat{\psi}_{i,MMRRPM} = \begin{bmatrix} \hat{b}_{i,p} \\ \hat{\lambda}_2 \hat{\gamma}_{i,NPR} \end{bmatrix}' \tag{3.28}$$

where $\hat{\lambda}_2$ is given in equation 3.32. Furthermore, the mean and variance are given by, respectively, $\bar{\psi}_{MMRRPM} = \frac{\sum_{i=1}^{m} \hat{\psi}_{i,MMRRPM}}{m}$ and $\hat{\Sigma}_{MMRRPM} = \frac{\sum_{i=1}^{m-1}(\hat{\psi}_{i+1,MMRRPM} - \hat{\psi}_{i,MMRRPM})(\hat{\psi}_{i+1,MMRRPM} - \hat{\psi}_{i,MMRRPM})'}{2(m-1)}$.

In semi-parametric techniques, $\lambda \in [0,1]$ is called the mixing parameter and it indicates the misspecification of the model. The proportions of parametric and non-parametric fits are controlled via it. λ is an unknown value and can be estimated from the data. Waterman et al. (2007) formulated a correlated data estimator for λ, and considered both PA and CS curves using MMRPM method, the estimators are given by, respectively:

$$\hat{\lambda}_{PA,MMRPM} = \frac{(\hat{y}_{i,-i}^{NP} - \hat{y}_{i,-i}^{P})'(y - \hat{y}_{PA}^{P})}{(\hat{y}_{PA}^{NP} - \hat{y}_{PA}^{P})'(\hat{y}_{PA}^{NP} - \hat{y}_{PA}^{P})} \tag{3.29}$$

$$\hat{\lambda}_{CS,MMRPM} = \frac{(\hat{y}_{i,-i}^{NP} - \hat{y}_{i,-i}^{P})'(y - \hat{y}_{CS}^{P})}{(\hat{y}_{CS}^{NP} - \hat{y}_{CS}^{P})'(\hat{y}_{CS}^{NP} - \hat{y}_{CS}^{P})} \tag{3.30}$$

where $\hat{y}_{i,-i}^{P}$ and $\hat{y}_{i,-i}^{NP}$ represent parametric and non-parametric fits of the ith cluster without the ith cluster.

By formulating Waterman et al.'s equation for PA and CS curves using MMRRPM method, the estimators are given by, respectively:

$$\hat{\lambda}_{PA,MMRRPM} = \frac{(\hat{r})'(y-\hat{y}_{PA}^{P})}{(\hat{r})'(\hat{r})} \tag{3.31}$$

$$\hat{\lambda}_{CS,MMRRPM} = \frac{(\hat{r})'(y-\hat{y}_{CS}^{P})}{(\hat{r})'(\hat{r})} \tag{3.32}$$

CHAPTER 4: PROPOSED PHASE II METHODS

As mentioned earlier, Phase II is used to monitor future observations by using the control limits obtained from the historical data in Phase I to determine if the process continues to be in-control or not. In this section, methods based on MCUSUM and MEWMA statistics are proposed to enhance the monitoring of LMM profiles in phase II. The performances of the proposed methods are compared using ARL and ATS (given in the appendix) criteria to decide which of the methods has the best performance.

4.1. General MCUSUM methods

The General Tabular CUSUM for detecting positive and negative shifts are given by, respectively:

$$C_i^+ = max(0, x_i - (\mu_0 + k) + C_{i-1}^+) \tag{4.1}$$

$$C_i^- = min(0, x_i - (\mu_0 - k) + C_{i-1}^-) \tag{4.2}$$

where x_i is the ith observation of the process, μ_0 is the target value and $k = \frac{\Delta}{2}$ is the reference value, where $\Delta = |\mu_1 - \mu_0|$ and μ_1 is the out-of-control value we are interested in detecting quickly. $C_i^+(0) = C_i^-(0) = 0$. The process is out-of-control if either C_i^+or C_i^- exceeds the decision interval H (positive or negative shifts, respectively), where H is chosen to guarantee that the ARL_0 (in-control ARL) reached a desired level.

Crosier (1988) proposed a MCUSUM control chart where the statistic is given by:

$$S_i = \begin{cases} 0 & d_i \le k \\ (S_{i-1} + Y_i - \mu_0)(1 - k/d_i) & d_i > k \end{cases} \tag{4.3}$$

where $S_0 = 0$, $d_i = [(S_{i-1} + Y_i - \mu_0)'\Sigma^{-1}(S_{i-1} + Y_i - \mu_0)]^{1/2}$, Σ is the variance-covariance matrix of Y and k is a predetermined constant. The MCUSUM signals if any of S_i's exceeds *H*. In this research, the MCUSUM is modified to get the proposed MCUSUM methods to detect step shifts in the slope for the parametric, non-parametric and semi-parametric techniques.

4.2. Proposed MCUSUM methods

4.2.1. Parametric estimated random effects (PMCUSUM)

$$S_{i,P} = \begin{cases} 0 & d_i \le k \\ \left(S_{i-1,\mathrm{P}} + \hat{b}_{i,P} - \bar{\bar{\mathrm{b}}}_P\right)(1 - k/d_i) & d_i > k \end{cases} \tag{4.4}$$

where $S_{0,\mathrm{P}} = 0$, k and d_i are explained in the previous section , $\hat{b}_{i,P}$ and $\bar{\bar{\mathrm{b}}}_P$ are the parametric estimated random effect and its mean vectors for the i^th profile, given in *Section 3.3.1*.

4.2.2. Non-Parametric estimated random effects (NPMCUSUM)

$$S_{i,NP} = \begin{cases} 0 & d_i \le k \\ (S_{i-1,NP} + \hat{\gamma}_{i,NP} - \bar{\bar{\gamma}}_{NP})(1 - k/d_i) & d_i > k \end{cases} \tag{4.5}$$

where $\mathrm{S}_{0,NP} = 0$, k and d_i are explained in the previous section , $\hat{\gamma}_{i,NP}$ and $\bar{\bar{\gamma}}_{NP}$ are the non-parametric estimated random effect and its mean vectors for the i^th profile, given in *Section 3.3.2*.

4.2.3. Non-Parametric estimated residuals random effects (NPRMCUSUM)

$$S_{i,NPR} = \begin{cases} 0 & d_i \leq k \\ \left(S_{i-1,NPR} + \hat{\gamma}_{i,NPR} - \bar{\bar{\gamma}}_{NPR}\right)(1 - k/d_i) & d_i > k \end{cases} \tag{4.6}$$

where $S_{0,NPR} = 0$, k and d_i are explained in the previous section , $\hat{\gamma}_{i,NPR}$ and $\bar{\bar{\gamma}}_{NPR}$ are the non-parametric residuals estimated random effect and its mean vectors for the i[th] profile, given in *Section 3.3.2*.

4.2.4. Semi-Parametric estimated random effects MMRPM (SPMCUSUM)

$$S_{i,MMRPM} = \begin{cases} 0 & d_i \leq k \\ \left(S_{i-1,MMRPM} + \hat{\psi}_{i,MMRPM} - \bar{\bar{\psi}}_{MMRPM}\right)(1 - k/d_i) & d_i > k \end{cases} \tag{4.7}$$

where $S_{0,MMRPM} = 0$, k and d_i are explained in the previous section , $\hat{\psi}_{i,MMRPM}$ and $\bar{\bar{\psi}}_{MMRPM}$ are the semi-parametric MRR1 estimated random effect and its mean vectors for the i[th] profile, given in *Section 3.3.3.*

4.2.5. Semi-Parametric estimated random effects MMRRPM (SPRMCUSUM)

$$S_{i,MMRRPM} = \begin{cases} 0 & d_i \leq k \\ \left(S_{i-1,MMRRPM} + \hat{\psi}_{i,MMRRPM} - \bar{\bar{\psi}}_{MMRRPM}\right)(1 - k/d_i) & d_i > k \end{cases} \tag{4.8}$$

where $S_{0,MMRRPM} = 0$, k and d_i are explained in the previous section , $\hat{\psi}_{i,MMRRPM}$ and $\bar{\bar{\psi}}_{MMRRPM}$ are the semi-parametric MRR2 estimated random effect and its mean vectors for the i[th] profile, given in *Section 3.3.3.*

4.3. General MEWMA methods

The general EWMA formula (Montgomery (2005)) used for detecting shifts is given by:

$$EWMA_i = wx_i + (1 - w)EWMA_{i-1} \quad (4.9)$$

where x_i is the i[th] observation of the process, w is the weighting factor (smoothing constant) that determines the importance of each observation x_i (0< $w \leq 1$). $EWMA_0 = \mu_0$, where μ_0 is the target value. The Upper Control Limit (UCL) and Lower Control Limit (LCL) of the process are given by:

$$UCL_{EWMA,i,} = \mu_0 + L\sigma\sqrt{\frac{w}{(2-w)}(1-(1-w)^{2i})} \quad (4.10)$$

$$LCL_{EWMA,i} = \mu_0 - L\sigma\sqrt{\frac{w}{(2-w)}(1-(1-w)^{2i})} \quad (4.11)$$

where L is the width of the control limits. The process is said to be out-of-control if $EWMA_i > UCL$ or $EWMA_i < LCL$. Lowry et al. (1992) developed a multivariate EWMA (MEWMA) control chart, which is an extension to the univariate EWMA given in (4.9), where EWMA's are vectors given by:

$$MEWMA_i = WX_i + (I - W)MEWMA_{i-1} \quad (4.12)$$

where $MEWMA_0 = 0$, X_i is a vector of observations and $W = diag(w_1, w_2, \ldots, w_p)$,

$0 < w_i \leq 1, i = 1,2, \ldots, p$, where w's are parameters that regulate the magnitude of the smoothing, usually they are chosen to be small for quicker detection of small shifts (Prabhu and Runger (1997)). The MEWMA signals when any of the T_i^2's exceeds the desired control limits.

$$T_i^2 = (MEWMA_i)'\Sigma_{MEWMA_i}^{-1}(MEWMA_i) \quad (4.13)$$

where $\Sigma_{EWMA_i}^{-1}$ is the inverse of the covariance matrix of $MEWMA_i$, if $w_1 = w_2 = \ldots = w_p$, then: $\Sigma_{MEWMA_i} = \frac{w}{2-w}[1-(1-w)^{2i}]\Sigma_x$, as $i \to \infty$: $\Sigma_{MEWMA_i} = \frac{w}{2-w}\Sigma_x$. If $w_1 \neq w_2 \ldots \neq w_p$, then: $\Sigma_{MEWMA_i}(k, L) = w_k w_L \frac{[1-(1-w_k)^i(1-w_L)^i]}{(w_k+w_L-w_k w_L)}\sigma_{kL}$. More details regarding how to design the MEWMA chart are found in Prabhu and Runger (1997). The MEWMA formulas are modified to get the proposed MEWMA for the parametric, non-parametric and semi-parametric techniques.

4.4. Proposed MEWMA methods

4.4.1. Parametric estimated random effects (PMEWMA)

$$MEWMA_{i,P} = W\hat{b}_{i,P} + (I - W)MEWMA_{i-1,P} \quad (4.14)$$

$$T_{i,P}^2 = (MEWMA_{i,P})'\Sigma_{MEWMA_i,P}^{-1}(MEWMA_{i,P}) \quad (4.15)$$

where $MEWMA_{0,P} = 0$, $\hat{b}_{i,P}$ is the estimated parametric random effects vector for the ith profile given in *Section 3.3.1*, W and $\Sigma_{MEWMA_i,P}^{-1}$ are explained previously.

4.4.2. Non-Parametric estimated random effects (NPMEWMA)

$$MEWMA_{i,NP} = W\hat{\gamma}_{i,NP} + (I - W)MEWMA_{i-1,NP} \quad (4.16)$$

$$T_{i,NP}^2 = (MEWMA_{i,NP})'\Sigma_{MEWMA_i,NP}^{-1}(MEWMA_{i,NP}) \quad (4.17)$$

where $MEWMA_{0,NP} = 0$, $\hat{\gamma}_{i,NP}$ is the estimated non-parametric random effects vector for the ith profile given in *Section 3.3.2*, W and $\Sigma_{MEWMA_i,NP}^{-1}$ are explained previously.

4.4.3. Non-Parametric estimated residuals random effects (NPRMEWMA)

$$MEWMA_{i,NPR} = W\hat{\gamma}_{i,NPR} + (I - W)MEWMA_{i-1,NPR} \quad (4.18)$$

$$T^2_{i,NPR} = (MEWMA_{i,NPR})'\Sigma^{-1}_{MEWMA_{i,NPR}}(MEWMA_{i,NPR}) \quad (4.19)$$

where $MEWMA_{0,NPR} = 0, \hat{\gamma}_{i,NPR}$ is the estimated non-parametric residuals random effects vector for the i^{th} profile given in *Section 3.3.2*, W and $\Sigma^{-1}_{MEWMA_{i,NPR}}$ are explained in the previous section.

4.4.4. Semi-Parametric estimated random effects MMRPM (SPMEWMA)

$$MEWMA_{i,SP} = W\hat{\psi}_{i,MMRPM} + (I - W)MEWMA_{i-1,SP} \quad (4.20)$$

$$T^2_{i,SP} = (MEWMA_{i,SP})'\Sigma^{-1}_{MEWMA_{i,SP}}(MEWMA_{i.SP}) \quad (4.21)$$

where $MEWMA_{0,SP} = 0, \hat{\psi}_{i,MMRPM}$ is the estimated semi-parametric MRR1 random effects vector for the i^{th} profile given in *Section 3.3.3*, W and $\Sigma^{-1}_{MEWMA_{i,SP}}$ are explained in the previous section.

4.4.5. Semi-Parametric estimated random effects MMRRPM (SPRMEWMA)

$$MEWMA_{i,SPR} = W\hat{\psi}_{i,MMRRPM} + (I - W)MEWMA_{i-1,SPR} \quad (4.22)$$

$$T^2_{i,SPR} = (MEWMA_{i,SPR})'\Sigma^{-1}_{MEWMA_{i,SPR}}(MEWMA_{i.SPR}) \quad (4.23)$$

where $MEWMA_{0,SPR} = 0, \hat{\psi}_{i,MMRRPM}$ is the estimated semi-parametric MRR2 random effects vector for the i^{th} profile given in *Section 3.3.3*, W and $\Sigma^{-1}_{MEWMA_{i,SPR}}$ are explained in the previous section.

CHAPTER 5: MONTE-CARLO SIMULATION

Comprehensive Monte-Carlo simulations are performed for generating correlated and uncorrelated data sets based on the adapted model from literature for comparing the proposed techniques to the parametric approach. These datasets are utilized to fit parametric regression estimation, two non-parametric regression estimations and two semi-parametric regression estimations for monitoring profiles and residuals obtained from linear mixed models at Phase II. SAS and R programs are used to carry out these analyses.

5.1. Simulation models

The data for cluster-specific (CS) is generated from the given model:

$$y_{ij} = (5 + b_{i1})x_{ij} + (2 + b_{i2})(x_{ij} - 5.5)^2 + \gamma\left[10\sin\left(\frac{\pi(x_{ij}-1)}{2.25}\right) + b_{i3}\right] + \epsilon_{ij} \quad (5.1)$$

$$i = 1,2, \dots, m \qquad j = 1,2, \dots, n_i$$

The profile average (PA) model is represented by:

$$y_{ij} = 5x_{ij} + 2(x_{ij} - 5.5)^2 + \gamma\left[10\sin\left(\frac{\pi(x_{ij}-1)}{2.25}\right)\right] + \epsilon_{ij} \quad (5.2)$$

$$i = 1,2, \dots, m \qquad j = 1,2, \dots, n_i$$

The model in (5.1) is adapted from Waterman (2002) and Abdelsalam (2009) for generating profile datasets from an assumed quadratic model given by:

$$y_{ij} = (\beta_0 + b_{i0}) + (\beta_1 + b_{i1})x_{ij} + (\beta_2 + b_{i2}){x_{ij}}^2 + \epsilon_{ij} \quad (5.3)$$

y_{ij} is simulated response variable of the jth observation of the ith profiles at the point x_{ij}. X regressor takes the integer values from 1 to 10. The b_{i0} is assumed equal to zero and b_{i1}, b_{i2} and b_{i3} are the independent random effects considered in this simulation study with $N(0,0.5)$ distribution.

The γ and *sin* function are the model misspecification amount and deviation from the quadratic model, respectively. Where five values for model misspecification are considered*: (γ =0, 0.25, 0.5, 0.75, 1)* for different profile sizes *(m=300, 600)* with the different number of observations *(n=10, 20)*.

ϵ_{ij} are the random errors with $N(0, R_i)$ distribution, where R_i is the variance-covariance matrix, $R_i = \sigma^2 I$ when the data is uncorrelated and with σ^2 *=16* (error variances), while for correlated data, R_i takes the first order auto-correlation matrix form AR(1). In this simulation, three significant different autocorrelation coefficients are considered (suggested by Waterman (2002)): a ρ *=0.2* for weak auto-correlation, a ρ *=0.5* for intermediate auto-correlation and a ρ *=0.8* for strong auto-correlation.

The upper control limits are obtained by fixing the in-control ARL_0, ARL_0 is known that it follows a geometric distribution with mean=1/p, where p=0.005 to give an ARL_0=200. By simulation, the ARL_0 values were chosen to be *200* and replicated under different shift sizes *(δ=0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.50)* in the slope. For simplicity, the smoothing parameters within each MEWMA control chart were selected to be equal ($w_1 = w_2 = \dots = w_p$), and in our numerical study; they were chosen to be *0.2* as was chosen by Kim et al. (2003), Narvand et al. (2013). For MCUSUM control charts, one selects the constant *k* to be half the amount of the shift that wants to be detected, and thus *k* was selected to be *0.5* for all control charts.

5.2. Simulation Studies

As mentioned before, different amounts of misspecification are considered (γ =0, 0.25, 0.5, 0.75, 1), the PA model is plotted, considering these different misspecification levels for *m*=300 and *n*=10. As it can be seen from the graph below *(Figure 5.1)*, γ =0 represents no model misspecification and it is the true probability model, as the amount of misspecification increases, the difference between the PA and the true probability model increases.

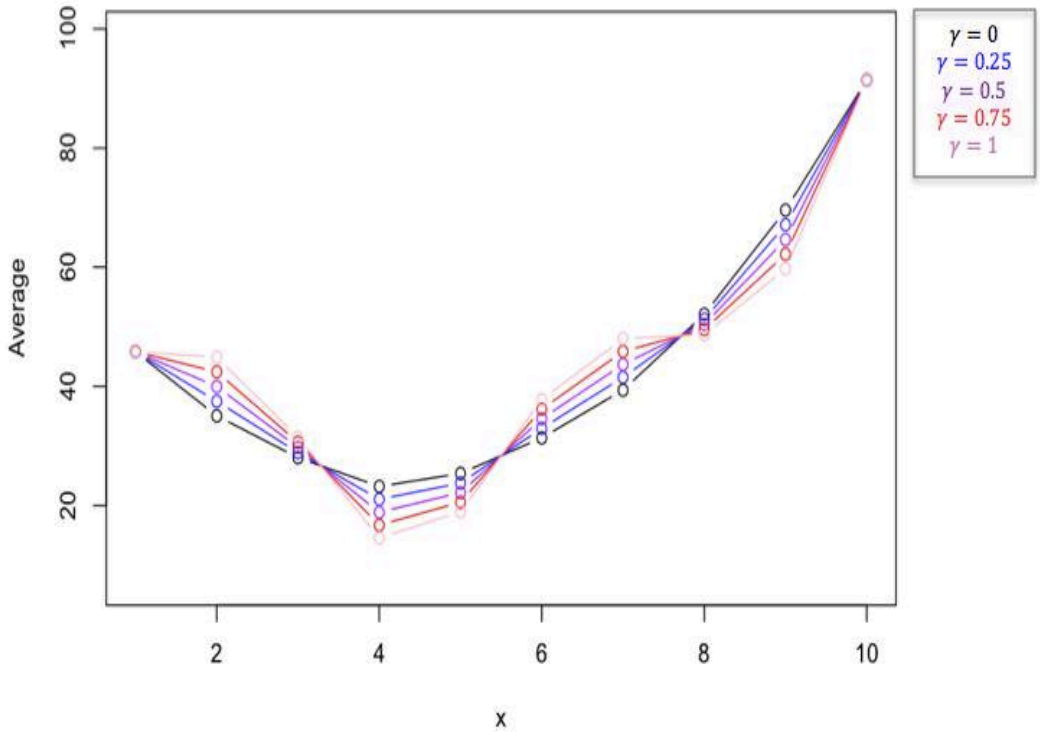

Figure 5.1. PA curves with different model misspecification levels ($\boldsymbol{\gamma}$), m=300 and n=10

All profile averages (PAs) and cluster specifics (CSs) are estimated using the parametric, non-parametric and semi-parametric techniques mentioned before, each PA and each CS has one fixed effect and one random effect. The estimations are done for 10000 simulation runs, as it is considered the best value for runs in terms of getting sufficient precision and saving computational time. The precision is examined using simulated integrated mean square error (SIMSE) for uncorrelated error terms and $\gamma = 0$ (no model misspecification) considering the different number of runs; where SIMSE is a goodness of fit measure for the estimated models given by:

$$SIMSE = \frac{1}{m}\sum_{i=1}^{m}(y_{CS,i} - \hat{y}_{CS,i})'(y_{CS,i} - \hat{y}_{CS,i}) \quad (5.4)$$

As it can be seen from the graph below *(Figure 5.2)*, as the simulation runs number increases, the standard error for the integrated mean square error decreases for each fitted model; after 10000 simulation runs, the standard errors are almost stable and therefore, it was chosen to be used in this simulation study.

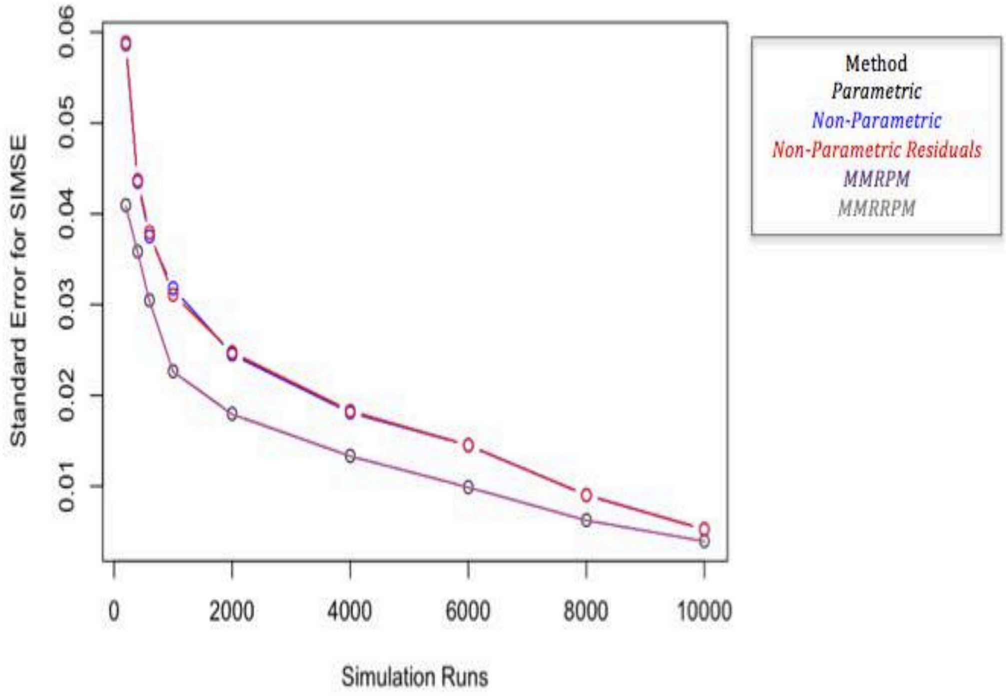

Figure 5.2. Standard Errors for SIMSE at different runs with no model misspecification $(\gamma = 0), m = 300$ and n=10

5.3. SIMSE results

The SIMSE was carried out for the fitted profiles using the different techniques mentioned before, considering multiple combinations of the different number of profiles *(m=300, 600)* with a different number of observations *(n=10, 20)* among all misspecification levels *(γ =0, 0.25, 0.5, 0.75, 1)* at varying correlations *(ρ =0, 0.2, 0.5, 0.8)*. The averages of SIMSE are recorded with their standard errors.

Table 5.1. SIMSE and (Standard Errors) for Different m,n and γ at No Correlation (ρ =0)

m	n	γ	P	NP	NPR	MMRPM	MMRRPM
300	10	0.00	2.45 (0.44)	3.93 (0.58)	3.95 (0.59)	2.45 (0.44)	2.45 (0.44)
		0.25	5.40 (0.45)	6.50 (0.59)	6.52 (0.60)	5.33 (0.45)	5.33 (0.45)
		0.50	14.36 (0.51)	14.25 (0.64)	14.26 (0.65)	13.70 (0.48)	13.70 (0.48)
		0.75	29.67 (0.73)	28.48 (0.73)	28.49 (0.74)	27.68 (0.58)	27.68 (0.59)
		1.00	51.70 (0.85)	48.31 (0.84)	48.31 (0.85)	48.65 (0.72)	48.64 (0.73)
300	20	0.00	0.83 (0.24)	2.89 (0.44)	2.16 (0.40)	0.83 (0.24)	0.83 (0.24)
		0.25	3.83 (0.26)	5.73 (0.46)	4.98 (0.43)	3.82 (0.36)	3.80 (0.26)
		0.50	12.85 (0.31)	14.25 (0.50)	13.49 (0.53)	12.83 (0.50)	12.64 (0.34)
		0.75	28.01 (0.43)	28.21 (0.55)	28.22 (0.67)	27.88 (0.55)	27.58 (0.48)
		1.00	49.45 (0.61)	45.37 (0.59)	45.58 (0.81)	45.97 (0.59)	45.33 (0.66)
600	10	0.00	2.49 (0.32)	4.06 (0.45)	4.07 (0.46)	2.49 (0.32)	2.49 (0.32)
		0.25	5.44 (0.32)	6.65 (0.48)	6.67 (0.48)	5.38 (0.33)	5.38 (0.33)
		0.50	14.40 (0.36)	14.43 (0.52)	14.44 (0.53)	13.81 (0.38)	13.81 (0.38)
		0.75	29.68 (0.51)	28.95 (0.59)	28.91 (0.59)	27.33 (0.48)	27.33 (0.48)
		1.00	51.64 (0.83)	48.54 (0.66)	48.54 (0.67)	48.86 (0.58)	48.85 (0.58)
600	20	0.00	0.86 (0.16)	2.22 (0.30)	2.23 (0.25)	0.86 (0.16)	0.86 (0.16)
		0.25	3.87 (0.17)	5.10 (0.31)	5.11 (0.35)	3.88 (0.17)	3.86 (0.17)
		0.50	12.99 (0.21)	13.68 (0.37)	13.66 (0.39)	12.90 (0.21)	12.74 (0.21)
		0.75	28.09 (0.37)	27.42 (0.40)	27.39 (0.46)	27.36 (0.27)	27.33 (0.30)
		1.00	49.54 (0.55)	45.91 (0.52)	45.86 (0.54)	45.59 (0.39)	45.06 (0.42)

As it can be seen from Table (5.1), as the sample size increases, the SIMSE results for all the techniques decrease because more observations mean higher precision. For the case when $n=10$ with varying profile sizes *($m=300, 600$),* the average SIMSE for both semi-parametric techniques (MMRPM & MMRRPM) are the smallest compared to the parametric (P), non-parametric (NP) and non-parametric residuals (NPR). When there is no model misspecification *($\gamma = 0$),* both averages of semi-parametric (MMRPM and MMRRPM) are close to the parametric (P) averages and as the model misspecification increases, the averages of the semi-parametric (MMRPM & MMRRPM) tend to increase and be closer to the non-parametric averages (NR & NPR). Furthermore, when the model misspecification level is one *($\gamma =1$)*, both the non-parametric averages (NR and NPR) have the least averages for small sample size. When $n=20$ with varying profile sizes *($m=300, 600$)*, both averages of semi-parametric (MMRPM & MMRRPM) are close to the parametric (P) averages with no model misspecification *($\gamma =0$),* as misspecification level (γ) increases, the semi-parametric (MMRRPM) tends to have the smallest average SIMSE compared to others. Also, both NP and NPR have almost the same results in terms of their SIMSE and their standard errors. It can be concluded that among most cases, MMRRPM gave the smallest average SIMSE, and it is the best option to be used compared to other techniques.

Table 5.2. SIMSE and (Standard Errors) for Different m,n and γ at Weak Auto-Correlation (ρ =0.2)

m	n	γ	P	NP	NPR	MMRPM	MMRRPM
300	10	0.00	3.18 (0.58)	5.19 (0.78)	5.24 (0.80)	3.18 (0.59)	3.18 (0.59)
		0.25	6.09 (0.58)	7.77 (0.78)	7.81 (0.80)	6.05 (0.59)	6.05 (0.59)
		0.50	14.92 (0.61)	15.52 (0.82)	15.55 (0.84)	14.49 (0.61)	14.49 (0.61)
		0.75	30.04 (0.76)	28.45 (0.89)	28.47 (0.91)	28.11 (0.69)	28.11 (0.69)
		1.00	51.82 (1.22)	46.57 (1.00)	46.57 (1.01)	46.74 (0.82)	46.74 (0.83)
300	20	0.00	1.19 (0.35)	3.72 (0.61)	3.03 (0.56)	1.19 (0.35)	1.19 (0.35)
		0.25	4.18 (0.36)	6.54 (0.64)	5.85 (0.60)	4.16 (0.36)	4.16 (0.36)
		0.50	13.16 (0.41)	15.06 (0.69)	14.35 (0.70)	13.14 (0.41)	13.02 (0.43)
		0.75	28.26 (0.49)	29.27 (0.75)	28.55 (0.84)	28.14 (0.50)	27.62 (0.65)
		1.00	49.61 (0.66)	49.18 (0.83)	48.43 (1.00)	49.19 (0.66)	47.86 (0.75)
600	10	0.00	3.25 (0.42)	5.38 (0.62)	5.42 (0.63)	3.25 (0.43)	3.25 (0.43)
		0.25	6.15 (0.42)	7.98 (0.64)	8.02 (0.65)	6.13 (0.43)	6.13 (0.43)
		0.50	14.99 (0.43)	15.76 (0.68)	15.79 (0.68)	14.62 (0.46)	14.62 (0.46)
		0.75	30.07 (0.53)	28.73 (0.73)	28.75 (0.73)	28.31 (0.55)	28.31 (0.55)
		1.00	51.78 (0.82)	46.88 (0.80)	46.88 (0.80)	47.00 (0.67)	47.00 (0.67)
600	20	0.00	1.23 (0.23)	4.07 (0.52)	3.14 (0.49)	1.23 (0.23)	1.23 (0.23)
		0.25	4.25 (0.23)	6.96 (0.53)	6.01 (0.49)	4.23 (0.23)	4.23 (0.23)
		0.50	13.25 (0.26)	15.54 (0.55)	14.57 (0.53)	13.23 (0.26)	13.13 (0.27)
		0.75	28.37 (0.33)	29.81 (0.61)	28.82 (0.59)	28.27 (0.33)	27.79 (0.35)
		1.00	49.73 (0.47)	49.78 (0.70)	48.76 (0.68)	49.34 (0.44)	48.11 (0.47)

From Table (5.2), It is observed that the averages for both semi-parametric techniques (MMRPM & MMRRPM) and the parametric technique (P) are the same when there is no model misspecification *(γ =0)* for different profile sizes and the different number of observations. For the case when *n=10,* regardless of the profile size*,* the averages for the non-parametric fit to the data (NP) are less compared to the non-parametric fit to the residuals (NPR), while it is less for NPR compared to NP when *n=20*. Furthermore, the averages of MMRRPM are smaller compared P, NP and NPR, both the semi-parametric have the same performances when n=10, but when *n=20,* MMRRPM outperformed MMRPM.

Table 5.3. SIMSE and (Standard Errors) for Different m,n and γ at Moderate Auto-Correlation (ρ =0.5*)*

m	n	γ	P	NP	NPR	MMRPM	MMRRPM
300	10	0.00	4.82 (0.95)	7.85 (1.24)	7.94 (1.27)	4.82 (0.95)	4.82 (0.95)
		0.25	7.64 (0.93)	10.42 (1.24)	10.51 (1.26)	7.63 (0.93)	7.63 (0.93)
		0.50	16.25 (0.76)	18.17 (1.26)	18.24 (1.28)	16.06 (0.90)	16.06 (0.90)
		0.75	30.99 (0.88)	31.11 (1.30)	31.16 (1.32)	29.84 (0.93)	29.84 (0.93)
		1.00	52.27 (1.15)	49.22 (1.37)	49.25 (1.38)	48.74 (1.05)	48.74 (1.05)
300	20	0.00	2.18 (0.63)	5.78 (1.84)	5.24 (1.00)	2.18 (0.63)	2.18 (0.63)
		0.25	5.13 (0.64)	8.60 (1.88)	8.05 (1.03)	5.12 (0.64)	5.12 (0.64)
		0.50	14.03 (0.65)	17.12 (1.95)	16.54 (1.12)	14.01 (0.66)	13.96 (0.67)
		0.75	28.99 (0.71)	31.32 (1.04)	30.71 (1.25)	28.89 (0.72)	28.62 (0.76)
		1.00	50.13 (0.84)	51.22 (1.14)	50.58 (1.41)	49.80 (0.85)	48.99 (0.93)

m	n	γ	P	NP	NPR	MMRPM	MMRRPM
600	10	0.00	4.97 (0.73)	8.17 (1.03)	8.26 (1.04)	4.97 (0.73)	4.97 (0.73)
		0.25	7.82 (0.71)	10.78 (1.04)	10.87 (1.06)	7.80 (0.71)	7.80 (0.71)
		0.50	16.41 (0.66)	18.57 (1.06)	18.65 (1.08)	16.26 (0.69)	16.26 (0.69)
		0.75	31.12 (0.64)	31.54 (1.09)	31.60 (1.11)	30.11 (0.74)	30.11 (0.74)
		1.00	52.33 (0.80)	49.70 (1.14)	49.73 (1.15)	49.11 (0.85)	49.11 (0.85)
600	20	0.00	2.23 (0.42)	6.20 (1.48)	5.40 (1.45)	2.23 (0.43)	2.23 (0.43)
		0.25	5.22 (0.42)	9.09 (1.50)	8.27 (1.55)	5.21 (0.42)	5.21 (0.42)
		0.50	14.16 (0.43)	17.68 (1.52)	16.83 (1.58)	14.14 (0.43)	14.10 (0.44)
		0.75	29.14 (0.46)	31.96 (1.55)	31.08 (1.63)	29.07 (0.46)	28.83 (0.46)
		1.00	50.31 (0.59)	51.93 (1.59)	51.01 (1.66)	50.02 (0.54)	49.29 (0.54)

The output of the averages of SIMSE at moderate auto-correlation *(ρ =0.5)* is given in Table (5.3), at all no misspecification *(*$\gamma = 0$) cases, the parametric (P), MMRPM and MMRRPM had the least averages, as the model misspecification increases, the averages of SIMSE of MMRRPM among all techniques had the least values. Moreover, for MMRPM, it had almost the same results of MMRRPM, except when there is a moderate to high misspecification levels for *n=20* with different profile sizes. Furthermore, the standard errors of both MMRPM and MMRRPM decrease when the misspecification levels are low to moderate, while increase when the misspecification levels increase from moderate to high.

Table 5.4. SIMSE and (Standard Errors) for Different m,n and γ at Strong Auto-Correlation (ρ =0.8)

m	n	γ	P	NP	NPR	MMRPM	MMRRPM
300	10	0.00	7.56	11.61	11.73	7.56	7.56
			(1.61)	(2.22)	(2.25)	(1.61)	(1.61)
		0.25	10.32	14.19	14.30	10.32	10.32
			(1.57)	(2.21)	(2.24)	(1.57)	(1.57)
		0.50	18.72	21.94	22.03	18.64	18.64
			(1.47)	(2.21)	(2.24)	(1.50)	(1.50)
		0.75	33.08	34.88	34.94	32.46	32.47
			(1.35)	(2.23)	(2.25)	(1.47)	(1.48)
		1.00	53.80	52.99	53.03	51.50	51.51
			(1.35)	(2.26)	(2.27)	(1.55)	(1.56)
300	20	0.00	4.56	10.16	9.87	4.55	4.56
			(1.19)	(2.04)	(2.07)	(1.19)	(1.19)
		0.25	7.45	12.98	12.68	7.44	7.46
			(1.18)	(2.09)	(2.09)	(1.18)	(1.18)
		0.50	16.21	21.49	21.16	16.19	16.20
			(1.15)	(2.16)	(2.15)	(1.16)	(1.16)
		0.75	30.90	35.70	35.31	30.85	30.79
			(1.13)	(2.16)	(2.14)	(1.14)	(1.17)
		1.00	51.66	55.59	55.14	51.49	51.19
			(1.16)	(2.37)	(2.36)	(1.18)	(1.24)
600	10	0.00	7.95	12.15	12.28	7.95	7.95
			(1.41)	(2.87)	(2.90)	(1.41)	(1.41)
		0.25	10.71	14.77	14.89	10.71	10.71
			(1.38)	(2.88)	(2.90)	(1.38)	(1.38)
		0.50	19.09	22.56	22.66	19.05	19.05
			(1.30)	(2.88)	(2.91)	(1.32)	(1.32)
		0.75	33.41	35.54	35.61	32.91	32.92
			(1.16)	(2.89)	(2.91)	(1.29)	(1.29)
		1.00	54.05	53.70	53.74	52.03	52.04
			(1.07)	(2.91)	(2.92)	(1.34)	(1.34)
600	20	0.00	4.66	10.69	10.12	4.65	4.66
			(1.92)	(2.06)	(2.09)	(1.92)	(1.92)
		0.25	7.61	13.57	12.98	7.59	7.61
			(1.90)	(2.07)	(2.09)	(1.91)	(1.90)
		0.50	16.39	22.15	21.52	16.38	16.39
			(1.88)	(2.10)	(2.11)	(1.88)	(1.88)
		0.75	31.12	36.42	35.74	31.08	31.04
			(1.84)	(2.13)	(2.15)	(1.86)	(1.87)
		1.00	51.91	56.39	55.64	51.77	51.52
			(1.82)	(2.17)	(2.19)	(1.86)	(1.90)

Table (5.4) below showed that when the observation size is $n=10$ with different profile sizes, the parametric, MMRPM and MMRRPM performed the same. The performance of NP is better than NPR when $n=10$ with different profile sizes, while when $n=20$, the performance of NPR is better. In all cases, when the model misspecification level increases, the semi-parametric techniques (MMRPM & MMRRPM) outperformed the parametric (P) and the non-parametric (NP & NPR) techniques. After checking the SIMSE of uncorrelated and all the AR(1) levels ($\rho = 0.0, 0.2, 0.5, 0.8$), we can say that both semi-parametric methods (MMRPM & MMRRPM) are close to the parametric (P) method when there is no model misspecification ($\gamma = 0$), but as γ increases, their results get closer to the non-parametric (NP & NPR) techniques. Furthermore, for small samples *(n=10)*, both the non-parametric (NP & NPR) had the same performances and the semi-parametric (MMRPM & MMRRPM) had the same performances in terms of their average SIMSE and standard errors, but for large samples *(n=20)*, NPR outperformed NP and MMRRPM outperformed MMRPM. Also, we can see that the SIMSE estimates get larger as ρ increases, this is because the mean square error depends on both bias and variance, thus as ρ increases (variance increases), the SIMSE increases as well.

5.4. MEWMA & MCUSUM results

In this section, the average run length (ARL) and extra quadratic loss (EQL) criteria is computed to measure the performances of the charts, as mentioned before, there is an in-control average run length (ARL_0) and an out-of-control average run length (ARL_1). The value of ARL_0 is large and it represents the process before any shift, the control limits of MEWMA and MCUSUM charts (given in the tables below) are chosen to give the desired ARL_0 and in this study, we fixed it to be ARL_0=200, when the shift is inserted, the ARL_1 is computed, the chart with the smallest ARL_1 value is considered the best chart (bolded in tables) as it takes less samples to detect shifts. The values for ARL_1 are computed for uncorrelated and all the correlated cases *(ρ =0.0, 0.2, 0.5, 0.8)* under all misspecification levels (γ =0.0, 0.25, 0.50, 0.75, 1.00) for different profiles *(m=300, 600)* and samples *(n=10, 20),* the ARL results for each case for MEWMA and MCUSUM charts are given in the tables below. Standard deviation run length (SDRL) and Average Time to Signal (ATS) results are given in the Appendix.

Furthermore, extra quadratic loss (EQL) results are given in the tables below, where Reynolds Jr. and Stoumbos (2004) suggested using it. EQL measures the overall performances of the charts and given by:

$$\mathrm{EQL} = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 \mathrm{ARL}(\delta)\, d\delta$$

where ARL(δ) is the ARL value of a chart at shift δ. The best chart is the one that has the least EQL value.

Moreover, some graphical representations for the charts performances are given in *Figures 5.3-5.6*.

Table 5.5. MEWMA ARL & EQL for Un-Correlated (ρ =0.0) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300, n=10, l=40)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	170.14	178.97	177.16	161.39	147.41
	0.10	150.48	156.89	158.60	139.43	125.40
	0.15	136.62	141.03	140.65	119.35	108.29
	0.20	113.42	122.88	120.38	105.16	90.90
	0.25	95.78	106.50	108.35	87.71	78.45
	0.30	72.94	90.66	89.84	69.54	64.29
	0.50	37.20	48.70	48.25	35.55	31.25
EQL		5.05	6.15	6.11	4.75	4.26
0.25	0.00	199.38	174.75	175.30	151.77	149.07
	0.05	171.77	156.87	155.34	135.50	130.83
	0.10	147.49	143.03	142.64	120.81	119.62
	0.15	133.63	122.44	121.78	106.43	103.32
	0.20	112.67	100.09	103.64	90.88	89.95
	0.25	96.83	82.02	83.85	66.11	64.90
	0.30	70.98	57.76	58.98	47.24	45.28
	0.50	38.71	30.10	31.53	26.72	24.72
EQL		5.07	4.17	4.29	3.56	3.40
0.50	0.00	196.49	149.87	147.89	135.42	135.63
	0.05	169.05	132.03	131.20	120.23	119.05
	0.10	141.09	116.94	115.73	107.91	104.59
	0.15	127.97	103.22	102.48	90.50	86.32
	0.20	112.16	89.58	89.52	72.19	68.46
	0.25	95.35	72.33	70.66	50.37	49.64
	0.30	78.35	50.63	52.75	38.06	35.31
	0.50	40.37	24.00	23.98	20.13	19.49
EQL		5.29	3.53	3.54	2.80	2.68
0.75	0.00	196.63	136.74	134.99	125.86	123.47
	0.05	174.10	119.32	117.79	107.29	103.53
	0.10	158.73	102.79	100.79	92.69	90.03
	0.15	140.54	85.40	87.40	74.31	71.66
	0.20	123.28	69.25	70.31	53.63	52.31
	0.25	100.22	54.57	55.75	39.99	38.22
	0.30	78.09	38.08	37.34	26.77	24.73
	0.50	42.69	20.31	19.56	13.26	11.01
EQL		5.52	2.81	2.77	2.01	1.83
1.00	0.00	198.69	128.96	126.10	110.84	104.42
	0.05	169.46	104.55	105.60	96.16	90.54
	0.10	142.61	90.49	90.15	83.84	78.33
	0.15	127.69	75.18	77.18	59.67	54.21
	0.20	110.20	62.27	60.06	42.43	37.00

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
1.00	0.25	93.58	44.58	43.03	27.28	24.99
	0.30	70.60	29.89	28.11	15.04	11.79
	0.50	35.59	16.21	15.18	7.47	6.71
EQL		4.86	2.29	2.19	1.29	1.12
UCL		32.77	46.53	46.60	32.77	32.77

Table 5.6. MEWMA ARL & EQL for Un-correlated (ρ =0.0) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=20, l=40)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	173.80	176.44	176.39	162.93	158.93
	0.10	150.85	154.29	153.53	145.36	139.60
	0.15	134.84	140.28	141.98	127.74	122.76
	0.20	117.48	124.79	126.98	112.10	106.97
	0.25	98.35	106.03	105.54	92.30	87.84
	0.30	75.14	84.09	83.53	70.22	66.29
	0.50	43.80	51.52	49.10	38.62	34.85
EQL		5.46	6.14	6.02	5.00	4.66
0.25	0.00	199.89	179.27	178.37	155.17	153.49
	0.05	168.17	162.20	160.49	134.96	129.56
	0.10	152.06	142.42	140.75	115.32	114.21
	0.15	138.52	120.94	121.39	100.03	100.63
	0.20	112.63	103.96	105.36	87.94	84.21
	0.25	93.35	84.58	83.84	65.47	61.38
	0.30	76.89	65.71	64.54	46.71	42.66
	0.50	40.13	34.46	32.59	25.74	21.78
EQL		5.27	4.60	4.48	3.47	3.14
0.50	0.00	199.40	165.87	163.54	146.97	142.53
	0.05	168.27	136.03	135.90	124.96	120.67
	0.10	154.70	119.15	117.98	107.88	106.53
	0.15	137.07	100.83	101.43	89.83	85.23
	0.20	115.69	84.99	87.92	74.50	69.21
	0.25	98.97	70.43	73.18	58.05	55.90
	0.30	79.16	58.13	60.44	39.52	38.67
	0.50	44.77	30.46	31.09	21.88	16.68
EQL		5.60	3.99	4.10	2.98	2.65
0.75	0.00	199.46	147.11	146.84	130.16	133.49
	0.05	172.74	122.42	124.57	116.97	119.47
	0.10	150.13	103.59	105.49	97.90	100.44

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.75	0.15	132.34	87.43	88.45	80.84	84.20
	0.20	110.52	73.51	75.25	66.23	70.78
	0.25	98.91	59.10	56.34	53.90	55.62
	0.30	77.06	46.34	47.04	34.50	36.45
	0.50	46.07	27.51	25.34	20.91	21.72
EQL		5.58	3.41	3.31	2.75	2.85
1.00	0.00	196.79	138.17	135.01	112.30	110.09
	0.05	170.51	116.48	114.87	100.21	95.56
	0.10	149.47	95.30	93.48	82.12	79.53
	0.15	132.71	78.76	80.94	65.94	63.78
	0.20	118.17	63.11	67.51	47.35	45.17
	0.25	101.18	49.95	46.15	34.88	30.67
	0.30	80.59	37.20	35.98	20.59	19.59
	0.50	47.37	22.15	20.84	13.92	11.63
EQL		5.77	2.81	2.71	1.82	1.64
UCL		33.09	46.00	46.02	33.09	33.09

Table 5.7. MEWMA ARL & EQL for Un-Correlated (ρ =0.0) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600, n=10, l=70)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	175.06	183.06	180.52	154.00	158.31
	0.10	158.47	161.90	162.43	136.79	142.20
	0.15	141.16	148.39	146.86	122.23	130.27
	0.20	123.64	133.25	133.78	107.88	114.55
	0.25	105.07	114.34	111.15	95.74	100.28
	0.30	88.68	100.60	97.84	77.60	85.94
	0.50	44.20	61.73	63.91	38.37	42.59
EQL		5.87	7.13	7.16	5.14	5.62
0.25	0.00	197.09	157.03	156.02	140.49	146.48
	0.05	173.31	144.59	143.32	128.79	134.40
	0.10	155.67	129.54	127.54	115.96	120.62
	0.15	138.33	115.03	113.44	102.24	107.34
	0.20	119.88	102.83	100.58	88.96	94.77
	0.25	100.74	90.18	89.85	76.87	82.67
	0.30	88.07	77.78	76.81	63.63	67.13
	0.50	56.15	45.34	45.52	35.22	38.80

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
EQL		6.41	5.18	5.38	4.40	4.74
0.50	0.00	198.15	131.22	132.05	125.22	126.99
	0.05	170.42	119.40	121.59	100.49	100.90
	0.10	150.47	104.53	104.44	87.34	88.59
	0.15	128.59	93.82	92.83	70.39	72.30
	0.20	114.67	80.73	78.73	62.36	65.77
	0.25	95.51	69.91	69.40	52.42	56.75
	0.30	80.24	58.72	57.74	46.96	49.84
	0.50	49.20	36.05	36.00	25.32	27.92
EQL		5.80	4.22	4.19	3.17	3.41
0.75	0.00	195.53	120.48	120.52	112.49	115.12
	0.05	165.90	104.68	103.68	92.52	98.97
	0.10	142.53	92.51	90.30	80.11	84.68
	0.15	120.31	80.01	83.01	68.45	70.45
	0.20	104.63	71.06	72.06	55.31	60.40
	0.25	88.79	58.97	59.98	45.60	49.17
	0.30	73.81	46.43	44.35	35.22	38.15
	0.50	46.93	21.39	21.02	14.26	16.93
EQL		5.43	3.06	3.01	2.26	2.52
1.00	0.00	198.70	114.93	113.52	95.24	98.64
	0.05	162.65	100.26	99.12	83.40	87.66
	0.10	140.40	88.88	90.88	74.46	76.67
	0.15	125.75	76.47	78.19	61.35	64.40
	0.20	109.60	62.60	63.36	47.35	50.20
	0.25	95.86	48.95	47.94	35.25	38.21
	0.30	79.74	37.27	38.57	23.13	25.05
	0.50	40.29	15.59	15.49	9.21	12.23
EQL		5.31	2.46	2.48	1.62	1.85
UCL		31.58	45.13	45.10	31.58	31.58

Table 5.8. MEWMA ARL & EQL for Un-correlated (ρ =0.0) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600, n=20 , l=70)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	170.48	173.94	176.16	149.29	155.19
	0.10	148.49	150.50	151.38	124.78	131.93
	0.15	130.35	134.47	136.24	102.79	111.35

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.20	114.43	116.31	117.23	87.97	97.43
	0.25	98.65	102.82	101.24	73.00	81.82
	0.30	82.93	84.83	84.24	52.07	56.33
	0.50	39.30	44.72	43.84	21.55	30.34
EQL		5.38	5.74	5.69	3.45	4.10
0.25	0.00	198.02	159.35	164.09	141.87	151.57
	0.05	164.06	142.87	149.49	127.97	136.79
	0.10	147.84	125.29	127.22	114.77	124.76
	0.15	123.31	110.27	114.90	100.56	106.71
	0.20	108.30	98.15	99.48	87.36	90.47
	0.25	93.80	81.33	83.66	73.94	77.73
	0.30	79.35	65.73	66.16	57.22	60.21
	0.50	39.14	38.51	38.54	24.51	25.17
EQL		5.22	4.71	4.75	3.69	3.85
0.50	0.00	194.22	139.86	140.83	120.29	125.63
	0.05	170.18	125.59	126.44	107.14	105.73
	0.10	152.41	110.76	111.59	95.98	92.67
	0.15	125.89	98.10	98.84	80.83	80.74
	0.20	101.82	84.20	85.51	64.42	60.94
	0.25	86.21	70.27	71.48	39.29	34.79
	0.30	72.15	58.15	57.82	23.04	20.62
	0.50	38.69	34.72	34.13	13.69	11.78
EQL		4.98	4.18	4.16	2.01	1.81
0.75	0.00	199.27	122.29	122.01	108.32	104.72
	0.05	164.36	110.20	110.76	88.46	86.78
	0.10	147.05	100.84	100.61	74.23	73.54
	0.15	128.59	88.96	89.20	58.20	54.48
	0.20	109.98	76.23	76.75	48.13	42.40
	0.25	95.13	63.75	62.52	32.94	30.30
	0.30	71.56	50.94	49.82	20.68	19.97
	0.50	36.74	27.89	28.31	10.29	10.15
EQL		4.95	3.57	3.56	1.60	1.53
1.00	0.00	199.76	107.94	108.71	93.52	92.94
	0.05	172.21	95.77	95.35	77.49	76.88
	0.10	159.45	80.17	81.69	63.25	61.80
	0.15	127.88	67.78	67.67	50.27	46.73
	0.20	108.44	55.20	54.11	37.22	34.77
	0.25	93.39	43.09	37.15	26.02	26.59
	0.30	71.96	31.33	26.57	15.95	13.56
	0.50	38.83	16.99	14.29	7.67	7.16
EQL		5.06	2.30	2.01	1.24	1.15
UCL		31.10	45.32	45.45	31.10	31.10

Table 5.9. MCUSUM ARL & EQL for Un-Correlated (ρ =0.0) Profile Datasets for Different Model Misspecification lLvels (γ) and Shift Sizes (δ) (m=300, n=10 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	168.49	178.95	176.23	157.33	151.37
	0.10	147.29	152.54	150.78	134.27	126.31
	0.15	130.52	135.81	133.87	110.53	107.47
	0.20	109.44	114.05	116.98	97.02	94.96
	0.25	90.32	100.94	102.88	80.68	79.46
	0.30	75.23	80.78	83.90	65.43	63.19
	0.50	38.90	42.06	42.41	33.94	29.77
EQL		5.12	5.51	5.61	4.48	4.19
0.25	0.00	197.59	178.64	177.03	160.90	160.07
	0.05	170.16	153.31	150.51	131.74	136.73
	0.10	148.29	130.20	129.09	118.22	114.59
	0.15	130.54	118.98	115.53	104.62	99.75
	0.20	118.53	103.41	102.97	90.26	87.21
	0.25	96.59	89.29	91.08	78.02	73.64
	0.30	74.36	68.41	66.50	59.56	60.09
	0.50	41.28	35.49	35.81	28.32	27.54
EQL		5.29	4.72	4.69	3.99	3.91
0.50	0.00	199.49	163.24	159.94	150.89	148.06
	0.05	169.20	138.60	139.76	129.07	125.73
	0.10	150.05	122.28	120.48	110.80	106.59
	0.15	135.11	99.72	100.74	93.40	89.38
	0.20	113.85	80.95	85.69	74.25	71.90
	0.25	97.60	67.82	71.36	61.96	58.11
	0.30	78.07	54.13	52.04	47.56	45.47
	0.50	43.61	31.49	30.18	25.12	22.38
EQL		5.49	3.92	3.85	3.36	3.13
0.75	0.00	196.72	155.02	150.22	140.10	137.52
	0.05	163.40	122.45	123.36	114.00	113.54
	0.10	148.74	107.67	109.46	98.64	95.39
	0.15	136.51	95.43	94.46	76.80	70.11
	0.20	120.10	69.68	70.17	60.72	55.95
	0.25	99.43	50.50	48.43	43.66	40.35
	0.30	70.61	38.80	35.68	30.61	26.67
	0.50	40.10	21.25	20.51	15.62	13.42
EQL		5.19	2.88	2.76	2.28	2.02
1.00	0.00	199.22	138.10	136.60	124.92	119.93
	0.05	156.56	116.46	115.87	113.02	108.97
	0.10	137.01	97.37	100.79	92.92	87.82
	0.15	125.71	78.71	84.18	73.81	69.82

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.50	0.20	110.18	56.94	66.88	50.71	50.68
	0.25	97.12	44.46	47.23	34.65	31.67
	0.30	75.53	32.65	29.84	16.39	15.56
	0.50	37.09	17.07	16.15	5.23	4.76
EQL		5.06	2.39	2.36	1.33	1.26
UCL		7.94	15.01	15.11	7.94	7.94

Table 5.10. MCUSUM ARL & EQL for un-correlated (ρ =0.0) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=20 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	168.67	173.27	169.32	154.26	153.20
	0.10	145.14	150.11	144.40	140.38	138.40
	0.15	120.95	130.60	126.75	115.50	115.95
	0.20	104.68	112.67	109.82	99.42	94.59
	0.25	84.57	99.32	96.35	80.49	76.40
	0.30	71.58	78.22	77.40	64.50	62.29
	0.50	40.50	46.92	47.64	38.40	34.52
EQL		5.04	5.66	5.63	4.71	4.42
0.25	0.00	199.53	180.68	179.21	165.60	161.46
	0.05	170.11	157.38	154.74	140.69	135.73
	0.10	155.38	146.15	140.62	123.28	117.40
	0.15	141.44	122.60	121.84	105.49	99.59
	0.20	126.60	104.90	105.24	93.59	84.29
	0.25	105.69	90.72	89.51	80.26	70.52
	0.30	85.38	68.42	67.42	55.20	54.20
	0.50	42.37	34.80	32.70	29.40	22.23
EQL		5.72	4.72	4.58	3.98	3.48
0.50	0.00	198.40	155.30	153.93	140.53	137.52
	0.05	171.23	141.69	139.24	125.29	123.11
	0.10	154.60	128.59	127.83	110.22	106.24
	0.15	138.78	112.39	109.44	95.60	90.38
	0.20	125.11	93.78	95.72	79.14	75.33
	0.25	106.69	77.45	74.83	63.66	60.39
	0.30	85.37	60.37	58.58	47.57	42.57
	0.50	44.30	30.24	27.42	21.50	20.35
EQL		5.81	4.14	3.94	3.21	2.99

0.75	0.00	199.58	142.84	142.24	127.50	130.95
	0.05	170.12	122.37	123.74	110.08	112.63
	0.10	153.60	100.39	103.16	94.50	93.52
	0.15	138.59	85.35	86.32	74.28	76.73
	0.20	119.24	72.60	73.58	61.25	61.60
	0.25	101.77	55.41	53.89	43.49	44.69
	0.30	84.69	40.29	39.83	29.13	30.52
	0.50	42.60	21.40	22.83	17.19	16.84
EQL		5.65	2.93	2.99	2.32	2.34
1.00	0.00	198.81	137.84	136.83	113.99	112.60
	0.05	168.94	122.50	123.81	105.30	104.67
	0.10	149.05	101.49	104.79	91.53	89.30
	0.15	131.57	84.71	82.69	78.40	76.43
	0.20	110.22	65.69	64.27	62.27	60.49
	0.25	99.26	50.37	50.86	45.00	46.22
	0.30	80.11	34.33	33.27	28.50	25.75
	0.50	45.30	19.66	19.20	15.47	13.44
EQL		5.61	2.65	2.60	2.23	2.06
UCL		7.90	15.03	15.40	7.90	7.90

Table 5.11. MCUSUM ARL & EQL for uncorrelated (ρ =0.0) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600, n=10 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	172.71	174.20	176.31	144.36	150.95
	0.10	147.65	158.39	160.03	120.75	129.42
	0.15	118.24	136.28	142.89	101.91	108.55
	0.20	106.40	117.12	120.11	87.09	91.50
	0.25	91.65	98.44	101.22	70.47	76.73
	0.30	75.77	80.68	81.90	52.00	56.15
	0.50	36.45	37.98	42.38	28.73	29.08
EQL		4.98	5.30	5.60	3.78	3.97
0.25	0.00	199.34	148.15	150.15	137.11	141.29
	0.05	167.71	134.59	129.06	120.39	125.41
	0.10	153.89	115.85	112.18	100.53	105.47
	0.15	140.97	97.70	99.72	87.28	93.42
	0.20	126.91	80.21	85.94	69.10	77.10
	0.25	103.83	62.11	69.02	57.03	64.92
	0.30	78.11	49.71	52.94	41.48	48.13
	0.50	37.41	29.15	32.17	21.42	26.83

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
EQL		5.29	3.65	3.94	2.96	3.48
0.50	0.00	199.09	134.60	135.14	124.44	126.36
	0.05	186.82	118.40	118.64	103.28	106.14
	0.10	159.82	104.92	104.56	90.72	94.43
	0.15	141.02	90.50	91.44	77.56	80.30
	0.20	137.56	77.00	77.03	64.55	65.33
	0.25	110.18	61.58	62.72	47.07	52.95
	0.30	81.52	42.25	42.51	33.84	35.78
	0.50	42.26	25.36	25.45	16.63	17.68
EQL		5.71	3.24	3.26	2.43	2.58
0.75	0.00	195.53	128.31	129.47	124.12	125.59
	0.05	175.87	114.87	115.41	107.06	110.48
	0.10	161.76	99.49	99.53	95.25	97.62
	0.15	143.50	85.45	86.08	79.93	81.51
	0.20	124.09	70.26	70.85	57.72	60.34
	0.25	106.04	54.91	54.97	43.61	45.22
	0.30	84.06	33.67	33.85	27.38	30.08
	0.50	39.77	18.24	18.26	14.46	15.26
EQL		5.56	2.61	2.62	2.14	2.27
1.00	0.00	196.85	111.90	112.15	104.28	104.93
	0.05	177.08	99.92	100.33	89.25	92.04
	0.10	154.58	89.82	90.69	76.31	79.97
	0.15	135.66	77.57	78.58	64.26	64.80
	0.20	117.73	63.66	63.77	47.24	48.80
	0.25	96.84	48.54	49.13	32.48	34.76
	0.30	82.27	26.24	26.92	18.91	19.53
	0.50	40.08	16.29	16.31	10.58	10.98
EQL		5.43	2.25	2.27	1.58	1.64
UCL		7.04	14.60	14.66	7.04	7.04

Table 5.12. MCUSUM ARL & EQL for uncorrelated (ρ =0.0) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600, n=20 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	175.08	180.07	178.61	153.51	157.52
	0.10	157.40	164.13	164.27	137.53	140.11
	0.15	136.95	148.97	150.34	120.75	124.47

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.20	119.12	130.41	134.18	98.28	102.14
	0.25	100.60	114.90	118.26	84.37	88.45
	0.30	86.03	96.18	98.88	70.38	74.18
	0.50	40.98	50.28	52.53	36.83	38.24
EQL		5.59	6.46	6.67	4.79	5.00
0.25	0.00	198.97	160.49	158.64	143.88	145.11
	0.05	174.15	138.56	137.95	127.84	130.00
	0.10	155.56	124.90	125.46	105.79	108.10
	0.15	136.84	101.84	99.35	91.64	95.93
	0.20	118.22	86.37	85.65	75.96	79.83
	0.25	103.30	64.09	61.36	58.10	62.33
	0.30	87.35	50.99	49.84	43.87	44.12
	0.50	39.82	31.32	31.74	26.98	25.58
EQL		5.58	3.84	3.81	3.34	3.33
0.50	0.00	199.64	147.66	142.64	124.71	126.45
	0.05	170.95	125.58	123.94	106.83	105.54
	0.10	150.16	104.43	103.54	94.18	90.75
	0.15	136.54	90.27	89.05	80.98	76.52
	0.20	121.91	75.37	72.95	66.45	63.23
	0.25	108.23	52.04	56.60	50.37	47.10
	0.30	88.72	39.84	40.29	31.99	30.94
	0.50	36.27	23.37	24.53	18.61	17.79
EQL		5.47	3.03	3.11	2.53	2.42
0.75	0.00	196.51	133.48	128.73	113.97	115.46
	0.05	169.59	114.33	112.50	100.09	97.54
	0.10	153.16	99.70	97.03	88.18	85.65
	0.15	136.88	86.77	85.98	74.48	70.73
	0.20	119.91	70.03	70.65	59.10	57.46
	0.25	91.09	54.09	52.69	42.58	43.06
	0.30	70.05	36.23	37.85	25.35	24.92
	0.50	38.23	15.32	16.59	13.36	11.14
EQL		5.04	2.52	2.61	2.02	1.88
1.00	0.00	193.76	117.49	115.59	92.86	94.43
	0.05	171.94	98.93	97.58	80.84	78.46
	0.10	149.82	85.56	83.28	66.15	65.61
	0.15	135.59	70.82	69.68	53.20	52.59
	0.20	119.25	57.26	56.39	39.10	40.48
	0.25	100.38	40.08	37.45	26.98	24.44
	0.30	83.48	25.42	21.93	17.57	15.24
	0.50	42.91	14.56	12.12	10.95	8.70
EQL		5.62	2.04	1.82	1.47	1.29
UCL		7.23	14.40	14.10	7.23	7.23

The tables above gave the out-of-control ARL_1 results for uncorrelated *(ρ =0.0)* profiles, *Tables (5.5 - 5.8)* represent the MEWMA results and *Tables (5.9 - 5.12)* represent the MCUSUM results for parametric, non-parametric and semi-parametric techniques. As it can be seen from all the tables above, when there is no model misspecification *(γ=0.0)*, the parametric technique is better than the non-parametric techniques in terms of their ARL_1 values, but the semi-parametric techniques (MMRPM and MMRRPM) provided better results than the parametric since their values are the least. As the model misspecification level starts increasing (*γ=0.25, 0.50, 0.75, 1.00),* the non-parametric and the semi-parametric approaches showed higher abilities to detect all amounts of shifts compared to the parametric approach. The two non-parametric have almost the same abilities in detecting shifts as their ARL_1 values are close to each other, yet, the semi-parametric MEWMA and MCUSUM charts are superior compared to the non-parametric charts. In almost all cases, when m=300, the MMRRPM results of both MEWMA and MCUSUM charts are better compared to MMRPM regardless to the sample size, while when m=600, the performance of MMRPM is better. Also, as we can notice, the MEWMA and the MCUSUM charts gave almost the same results in detecting different shifts (MEWMA≈MCUSUM). Furthermore, based on the EQL results obtained, both MMRPM & MMRRPM had the lowest EQL values and thus the best. Both ATS and SDRL criteria (given in the Appendix) gave the same conclusions as ARL. In addition, ARL graphs (*Figure 5.3)* for *m=300* and *n=10* profiles were plotted to show the performances of these charts at different misspecification levels, as it can be seen from the graphs, as model misspecification level increases, the proposed charts have superior performances compared to the classical parametric charts.

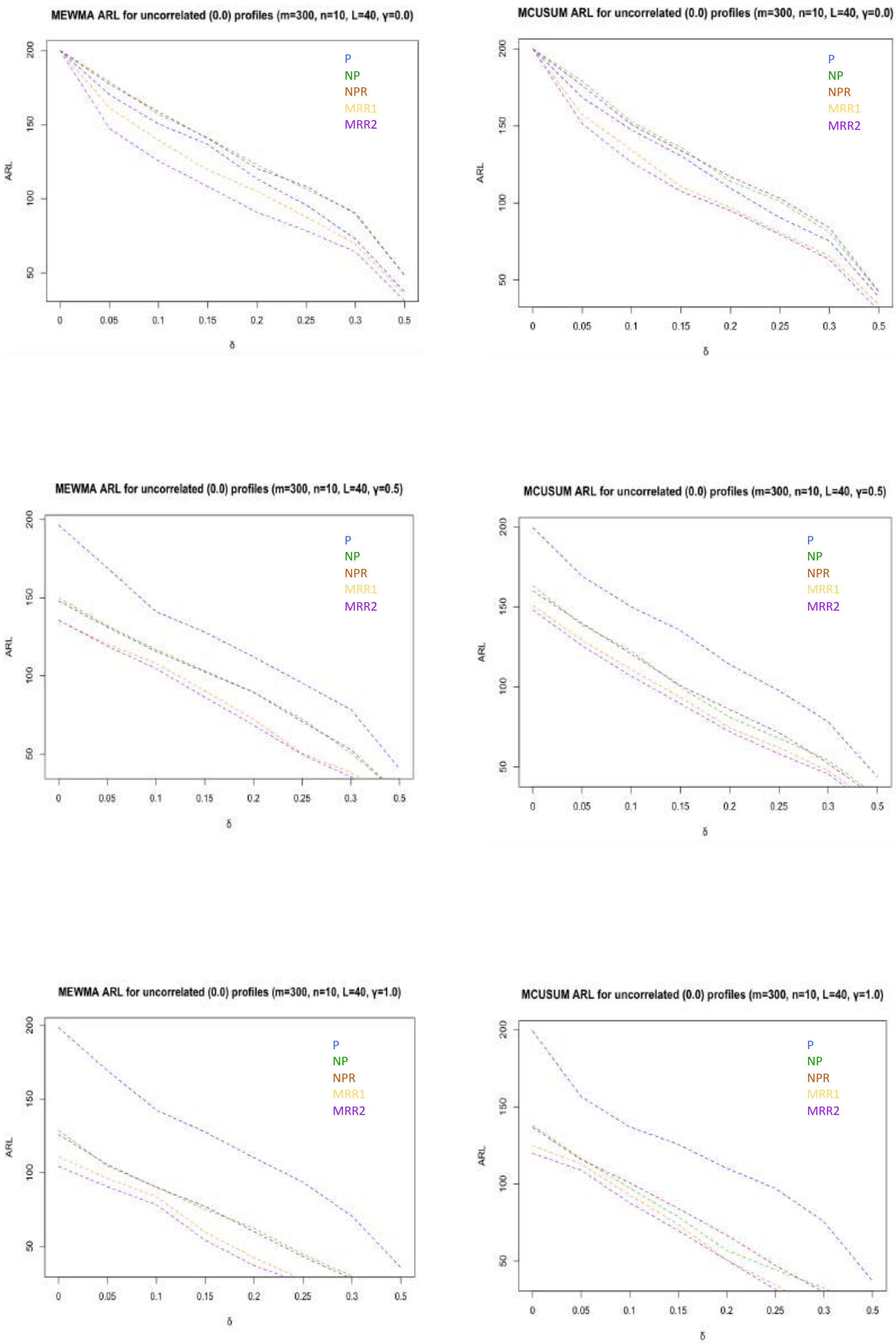

Figure 5.3. ARL graphs for uncorrelated (ρ =0.0) profiles

Table 5.13. MEWMA ARL & EQL for Weak Auto-Correlated (ρ =0.2) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=10 , l=40)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	173.31	178.20	177.24	163.76	161.69
	0.10	152.35	157.64	154.59	148.76	145.67
	0.15	140.07	145.29	143.07	132.53	129.44
	0.20	127.89	130.22	128.89	120.17	117.08
	0.25	107.08	119.76	118.20	99.36	94.27
	0.30	88.87	97.80	94.33	81.13	78.04
	0.50	52.38	60.22	58.89	44.20	40.19
EQL		6.31	7.01	6.84	5.62	5.30
0.25	0.00	196.64	156.15	153.52	145.17	140.29
	0.05	171.62	133.89	131.36	124.97	118.98
	0.10	150.63	121.71	117.19	114.57	103.67
	0.15	130.69	109.78	109.71	100.47	93.90
	0.20	119.44	91.39	91.38	83.86	79.36
	0.25	97.56	80.26	79.22	71.35	66.37
	0.30	83.89	66.47	65.98	59.61	52.71
	0.50	50.30	42.62	41.36	39.84	34.01
EQL		5.96	4.85	4.81	4.48	3.96
0.50	0.00	198.81	140.22	138.83	128.15	127.59
	0.05	168.71	117.02	115.09	105.39	104.20
	0.10	149.62	103.78	104.93	98.29	98.64
	0.15	127.92	93.70	90.20	79.30	87.98
	0.20	112.25	80.40	76.32	70.39	73.00
	0.25	94.47	70.09	67.18	62.99	60.10
	0.30	79.59	59.68	53.33	50.19	49.38
	0.50	47.96	34.28	32.57	26.73	25.93
EQL		5.71	4.16	3.89	3.44	3.39
0.75	0.00	198.31	124.66	123.55	110.28	109.59
	0.05	167.82	104.11	105.57	89.17	88.78
	0.10	148.24	92.48	93.65	75.18	73.52
	0.15	125.04	84.71	82.70	60.86	59.69
	0.20	108.27	70.79	69.41	54.71	49.99
	0.25	92.92	55.83	55.77	45.23	40.31
	0.30	77.45	43.83	44.31	34.71	30.28
	0.50	50.65	22.47	20.36	17.21	16.72
EQL		5.76	3.05	2.94	2.37	2.19
1.00	0.00	195.35	101.62	101.20	87.35	86.39
	0.05	168.91	90.96	90.95	75.70	73.42
	0.10	143.60	79.32	79.98	65.79	60.38
	0.15	120.59	68.86	70.49	52.77	49.47

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
	0.20	104.57	60.90	57.70	40.50	35.69
1.00	0.25	92.96	48.87	43.79	31.26	26.00
	0.30	75.47	38.43	30.80	19.99	18.73
	0.50	48.02	18.84	18.77	10.26	9.65
EQL		5.55	2.61	2.39	1.52	1.39
UCL		32.71	46.75	46.60	32.71	32.71

Table 5.14. MEWMA ARL & EQL for Weak Auto-Correlated (ρ =0.2) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=20 , l=40)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	175.05	178.71	178.77	160.26	159.52
	0.10	160.94	165.67	164.58	144.95	145.25
0.00	0.15	147.77	153.21	151.40	132.75	133.03
	0.20	130.29	136.09	134.88	119.99	120.12
	0.25	111.58	117.49	116.61	100.14	101.21
	0.30	92.12	98.68	96.79	86.11	86.90
	0.50	57.13	64.44	58.01	50.07	48.11
EQL		6.68	7.27	6.89	6.03	5.95
	0.00	197.89	166.71	166.74	145.62	147.52
	0.05	171.81	138.77	136.67	123.57	125.05
	0.10	154.03	120.81	115.35	110.41	107.74
0.25	0.15	143.77	104.60	100.94	99.48	96.30
	0.20	128.81	90.67	86.78	85.37	83.95
	0.25	107.41	79.66	70.60	70.29	66.42
	0.30	90.96	67.37	59.80	59.40	55.41
	0.50	56.90	43.41	38.44	38.01	35.36
EQL		6.59	4.93	4.43	4.38	4.12
	0.00	196.95	149.58	149.28	127.83	130.29
	0.05	165.75	126.40	125.53	106.76	109.48
	0.10	152.01	112.57	111.36	95.81	97.83
0.50	0.15	140.49	100.48	98.62	87.83	84.47
	0.20	125.10	87.64	85.58	79.55	72.37
	0.25	103.90	74.30	70.11	70.03	62.14
	0.30	87.25	63.24	54.31	57.44	53.70
	0.50	56.69	40.43	37.09	33.86	30.52
EQL		6.45	4.62	4.22	4.06	3.72
0.75	0.00	199.50	128.96	127.96	109.48	110.29

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.75	0.05	166.46	103.57	102.81	85.32	85.60
	0.10	150.29	92.11	90.28	72.34	70.24
	0.15	134.63	79.93	77.24	64.21	59.43
	0.20	120.59	66.58	65.13	50.56	46.54
	0.25	98.74	55.33	53.28	38.74	36.29
	0.30	84.50	42.92	40.30	25.29	23.19
	0.50	50.39	25.95	24.58	13.67	11.44
EQL		6.01	3.17	3.02	1.93	1.73
1.00	0.00	198.67	110.28	109.36	84.39	88.40
	0.05	164.70	92.66	90.26	65.83	70.47
	0.10	146.68	84.56	82.35	50.38	52.36
	0.15	130.68	73.76	71.23	43.46	44.46
	0.20	113.79	60.08	59.44	32.59	35.11
	0.25	91.19	45.41	44.23	23.33	24.28
	0.30	75.06	36.51	34.59	15.00	13.80
	0.50	48.28	18.42	17.52	6.31	7.06
EQL		5.61	2.54	2.43	1.09	1.12
UCL		32.85	46.15	46.22	32.85	32.85

Table 5.15. MEWMA ARL & EQL for Weak Auto-Correlated (ρ =0.2) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=10 , l=70)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	163.49	172.02	169.31	163.49	163.49
	0.10	134.28	149.70	146.83	134.28	134.28
	0.15	118.62	130.36	129.91	118.62	118.62
	0.20	94.69	111.32	109.00	94.69	94.69
	0.25	77.16	98.39	96.06	77.16	77.16
	0.30	60.79	80.67	79.85	60.79	60.79
	0.50	34.05	46.35	46.31	34.05	34.05
EQL		4.37	5.67	5.62	4.37	4.37
0.25	0.00	199.23	160.47	161.04	146.43	142.75
	0.05	165.00	138.12	139.56	130.09	128.95
	0.10	131.54	113.14	111.18	106.35	103.09
	0.15	120.20	98.46	95.96	92.26	90.71
	0.20	98.14	82.81	80.79	75.55	72.21
	0.25	76.30	65.62	65.60	59.84	50.11

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.25	0.30	62.76	50.98	51.91	41.92	36.34
	0.50	35.79	30.20	30.17	20.11	19.13
EQL		4.51	3.76	3.77	2.97	2.71
0.50	0.00	195.17	142.80	142.49	121.30	120.52
	0.05	166.10	127.76	127.76	108.99	106.04
	0.10	134.56	105.96	104.95	95.75	93.43
	0.15	116.30	92.23	91.95	80.62	77.42
	0.20	94.76	76.61	75.88	68.13	64.82
	0.25	78.07	58.53	57.61	53.83	45.46
	0.30	61.20	41.66	40.59	37.79	32.47
	0.50	34.48	24.52	24.49	17.96	16.26
EQL		4.40	3.18	3.14	2.66	2.38
0.75	0.00	194.30	126.47	123.45	109.20	104.94
	0.05	171.54	105.89	105.04	97.04	94.93
	0.10	147.37	93.28	92.28	84.81	81.70
	0.15	129.79	80.17	78.17	68.61	64.43
	0.20	111.23	67.62	65.92	54.37	50.35
	0.25	98.61	51.58	47.05	38.20	34.26
	0.30	71.34	38.41	35.38	25.18	20.39
	0.50	40.48	20.91	20.61	15.03	13.62
EQL		5.17	2.80	2.67	2.03	1.80
1.00	0.00	197.26	117.97	117.95	98.72	97.26
	0.05	173.73	102.87	97.10	82.80	79.51
	0.10	148.44	86.77	83.70	68.73	67.43
	0.15	126.69	73.96	71.87	55.61	54.50
	0.20	110.65	60.66	58.25	38.25	37.42
	0.25	93.04	46.28	45.19	25.11	23.38
	0.30	76.63	32.13	31.99	15.87	14.09
	0.50	39.79	15.35	15.27	7.24	6.59
EQL		5.21	2.30	2.26	1.24	1.15
UCL		31.81	45.00	44.90	31.81	31.81

Table 5.16. MEWMA ARL & EQL for Weak Auto-Correlated (ρ =0.2) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=20 , l=70)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	174.09	176.43	176.86	171.34	169.75

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.10	157.51	162.60	162.94	150.67	148.04
	0.15	140.20	145.43	146.39	138.75	139.68
	0.20	126.04	129.32	130.92	119.03	122.64
	0.25	111.63	110.82	114.93	108.93	105.62
	0.30	86.07	85.71	90.53	79.25	77.71
	0.50	39.28	43.83	46.07	36.19	36.89
EQL		5.61	5.86	6.11	5.25	5.24
0.25	0.00	192.07	161.44	165.42	141.87	146.66
	0.05	176.10	141.88	141.24	126.70	128.69
	0.10	145.39	125.30	123.56	111.30	116.61
	0.15	129.63	103.27	102.90	96.29	100.71
	0.20	105.62	86.31	86.65	80.91	79.93
	0.25	90.27	73.48	71.36	66.66	68.85
	0.30	76.81	60.65	55.13	42.99	45.08
	0.50	43.23	31.80	30.15	23.48	24.13
EQL		5.35	4.15	3.93	3.24	3.34
0.50	0.00	195.27	137.10	141.01	120.14	125.49
	0.05	169.55	119.28	124.64	109.87	107.47
	0.10	152.22	100.05	103.69	89.93	85.66
	0.15	124.95	85.71	87.82	75.61	72.39
	0.20	100.03	71.41	68.05	62.57	59.02
	0.25	88.56	56.46	52.45	49.20	45.71
	0.30	72.16	41.03	40.52	35.98	32.62
	0.50	39.17	24.43	26.18	17.62	16.66
EQL		5.01	3.10	3.15	2.53	2.36
0.75	0.00	193.44	114.58	113.03	98.38	95.08
	0.05	170.27	97.71	94.42	84.73	80.18
	0.10	149.73	83.02	82.76	70.51	67.97
	0.15	127.78	70.39	69.84	60.30	55.59
	0.20	105.38	56.35	57.84	49.23	45.50
	0.25	90.04	44.54	42.97	37.94	32.19
	0.30	71.84	30.13	29.32	25.68	24.03
	0.50	38.72	17.96	16.23	12.00	13.41
EQL		5.01	2.34	2.23	1.83	1.80
1.00	0.00	192.63	96.11	96.44	87.22	85.88
	0.05	186.01	80.33	80.15	75.17	73.89
	0.10	165.05	67.54	66.35	66.01	63.67
	0.15	129.91	60.75	58.93	54.77	50.44
	0.20	100.07	48.92	45.93	40.71	37.34
	0.25	86.04	35.84	30.92	26.71	23.25
	0.30	70.28	26.09	21.90	18.45	15.06
	0.50	36.05	13.37	11.36	10.90	9.69
EQL		4.82	1.89	1.65	1.49	1.31
UCL		31.15	45.58	45.50	31.15	31.15

Table 5.17. MCUSUM ARL & EQL for Weak Auto-Correlated (ρ =0.2) Profile Datasets for Different Model Misspecification levels (γ) and Shift Sizes (δ) (m=300 , n=10 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	169.02	176.21	174.96	161.99	160.89
	0.10	150.38	154.90	153.86	146.28	143.18
	0.15	138.39	139.45	138.50	129.19	125.09
	0.20	124.28	125.40	123.36	117.11	112.01
	0.25	103.89	112.75	113.94	96.74	95.65
	0.30	84.30	95.19	92.02	80.55	78.44
	0.50	48.74	56.46	54.84	43.21	42.35
EQL		5.98	6.68	6.52	5.52	5.39
0.25	0.00	195.90	149.08	149.02	139.78	138.63
	0.05	169.92	126.94	126.66	117.97	116.31
	0.10	149.32	114.14	113.53	109.89	107.94
	0.15	128.57	98.00	97.77	89.34	87.02
	0.20	115.49	85.35	83.94	77.12	76.27
	0.25	94.47	73.90	66.80	63.10	59.29
	0.30	82.86	60.73	55.58	52.98	48.79
	0.50	47.74	38.87	33.78	34.87	33.39
EQL		5.78	4.47	4.05	3.97	3.77
0.50	0.00	196.19	134.28	133.27	122.40	120.49
	0.05	167.72	112.27	111.12	103.57	102.78
	0.10	144.52	101.35	99.80	95.31	94.68
	0.15	124.51	89.82	87.92	80.42	79.80
	0.20	98.02	78.39	76.32	70.59	68.52
	0.25	82.49	65.94	63.49	59.79	57.89
	0.30	73.74	48.81	48.57	45.63	44.67
	0.50	44.66	30.74	28.71	24.74	23.50
EQL		5.26	3.69	3.55	3.22	3.11
0.75	0.00	198.81	121.09	121.06	108.94	106.46
	0.05	165.31	102.04	100.45	87.80	86.22
	0.10	144.00	89.05	87.05	78.96	77.08
	0.15	122.82	77.03	76.98	70.33	69.18
	0.20	104.78	69.60	68.80	52.34	50.16
	0.25	90.20	55.53	54.59	40.07	38.13
	0.30	75.27	42.42	35.42	30.67	30.22
	0.50	43.66	21.3	19.29	15.23	14.59
EQL		5.32	2.93	2.66	2.17	2.10
1.00	0.00	196.71	96.47	95.97	83.20	81.99
	0.05	166.62	85.60	85.58	74.29	73.46
	0.10	140.20	78.69	76.09	62.27	57.76

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
1.00	0.15	120.51	65.42	67.48	50.27	45.63
	0.20	103.91	58.22	55.66	39.07	35.31
	0.25	87.69	49.08	47.79	29.95	23.46
	0.30	73.24	35.28	33.27	18.87	15.35
	0.50	42.19	15.41	13.36	9.85	7.35
EQL		5.98	6.68	6.52	5.52	5.39
UCL		7.95	15.08	15.05	7.95	7.95

Table 5.18. MCUSUM ARL & EQL for Weak Auto-Correlated (ρ =0.2) Profile Datasets for Different Model Misspecification levels (γ) and Shift Sizes (δ) (m=300 , n=20 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	173.16	176.73	175.63	155.81	160.91
	0.10	157.83	162.03	161.34	140.44	148.74
	0.15	145.79	150.54	149.97	128.47	135.53
	0.20	127.18	133.54	132.36	115.29	118.27
	0.25	107.39	115.17	114.43	101.19	100.47
	0.30	90.16	94.72	93.39	85.16	86.91
	0.50	55.60	56.53	55.46	46.90	47.47
EQL		6.51	6.75	6.66	5.82	5.92
0.25	0.00	198.29	160.39	161.32	143.23	144.57
	0.05	169.39	136.15	135.52	123.20	130.62
	0.10	153.20	124.52	123.32	110.59	115.23
	0.15	142.34	111.03	110.39	95.80	100.48
	0.20	124.52	100.94	98.66	85.60	90.68
	0.25	106.53	87.93	86.30	70.59	79.59
	0.30	88.40	78.51	75.74	57.59	62.69
	0.50	53.52	47.13	45.38	31.24	36.36
EQL		6.34	5.48	5.31	3.99	4.46
0.50	0.00	199.57	146.49	145.52	125.21	125.42
	0.05	164.42	124.71	122.40	102.38	106.46
	0.10	150.39	110.62	109.63	90.98	100.64
	0.15	136.74	96.05	96.49	82.37	94.58
	0.20	124.53	84.19	85.51	74.57	84.79
	0.25	102.46	73.59	72.49	66.40	72.86
	0.30	85.60	63.49	60.21	54.21	60.68
	0.50	54.32	40.38	39.18	27.93	31.25

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
EQL		6.27	4.60	4.46	3.63	4.06
0.75	0.00	194.57	125.33	125.00	105.89	106.49
	0.05	165.49	100.14	98.59	83.47	94.31
	0.10	149.21	89.14	87.85	71.46	80.37
	0.15	133.57	75.23	72.34	60.11	68.46
	0.20	117.64	63.17	60.47	50.37	52.46
	0.25	94.48	52.12	50.39	35.63	40.53
	0.30	80.12	44.04	39.42	27.55	29.22
	0.50	53.47	25.64	24.01	15.73	18.63
EQL		5.98	3.13	2.91	2.05	2.31
1.00	0.00	196.53	107.40	106.20	90.57	92.21
	0.05	169.38	91.58	89.58	76.41	77.64
	0.10	146.45	82.36	80.27	64.22	65.79
	0.15	133.98	70.39	70.66	53.54	54.33
	0.20	114.06	58.98	59.37	42.89	43.32
	0.25	93.30	45.34	44.41	30.31	31.29
	0.30	76.21	34.73	32.98	19.12	19.31
	0.50	51.95	17.26	16.56	5.01	5.43
EQL		5.84	2.42	2.34	1.24	1.28
UCL		7.82	15.12	15.30	7.82	7.82

Table 5.19. MCUSUM ARL & EQL for Weak Auto-Correlated (ρ =0.2) Profile Datasets for Different Model Misspecification levels (γ) and Shift Sizes (δ) (m=600 , n=10 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	166.39	174.74	178.51	166.39	166.39
	0.10	135.02	152.49	155.28	135.02	135.02
	0.15	120.24	134.94	132.63	120.24	120.24
	0.20	101.30	115.09	114.75	101.30	101.30
	0.25	85.35	100.08	99.14	85.35	85.35
	0.30	64.11	83.77	85.96	64.11	64.11
	0.50	37.91	45.13	48.64	37.91	37.91
EQL		4.72	5.72	5.94	4.72	4.72
0.25	0.00	194.10	159.86	160.46	147.43	145.31
	0.05	172.40	140.71	139.10	128.65	125.56

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.25	0.10	142.00	114.35	112.14	107.59	105.50
	0.15	129.44	100.61	97.04	89.55	88.43
	0.20	103.27	84.19	80.53	75.36	75.24
	0.25	81.32	69.76	67.64	60.28	58.01
	0.30	65.01	55.16	54.50	46.87	44.41
	0.50	36.74	30.87	29.45	23.57	22.91
EQL		4.69	3.93	3.80	3.25	3.14
0.50	0.00	199.98	141.78	140.47	126.35	124.40
	0.05	171.39	128.73	126.22	106.39	104.75
	0.10	146.90	107.53	104.69	90.53	87.79
	0.15	123.23	86.08	84.94	76.53	74.67
	0.20	100.71	70.97	69.06	62.24	59.36
	0.25	78.81	57.07	54.49	50.06	45.98
	0.30	65.60	43.23	41.99	36.98	33.93
	0.50	34.60	22.62	20.12	17.76	16.57
EQL		4.56	3.07	2.89	2.57	2.39
0.75	0.00	197.95	129.96	128.32	105.56	103.94
	0.05	169.79	105.16	101.80	90.48	86.85
	0.10	150.77	87.03	87.80	77.48	73.88
	0.15	132.19	74.16	75.48	65.51	61.95
	0.20	107.81	60.51	62.47	52.54	49.90
	0.25	91.30	49.58	48.57	35.35	32.54
	0.30	68.46	34.26	32.84	23.14	20.34
	0.50	35.71	18.14	17.89	14.42	12.43
EQL		4.81	2.50	2.46	1.91	1.71
1.00	0.00	191.46	110.79	107.43	94.92	92.70
	0.05	166.91	96.97	94.10	80.02	77.70
	0.10	143.37	80.21	77.11	66.04	62.97
	0.15	125.94	68.19	65.64	52.06	49.86
	0.20	94.62	55.69	51.91	35.08	32.70
	0.25	78.54	42.41	38.52	22.81	21.53
	0.30	54.97	27.04	24.93	14.72	12.45
	0.50	30.15	16.41	16.03	8.02	7.13
EQL		4.08	2.17	2.05	1.21	1.09
UCL		7.12	14.62	14.35	7.12	7.12

Table 5.20. MCUSUM ARL & EQL for Weak Auto-Correlated (ρ =0.2) Profile Datasets for Different Model Misspecification levels (γ) and Shift Sizes (δ) (m=600 , n=20 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	168.24	171.88	170.15	167.06	161.92
	0.10	152.78	157.73	156.91	150.57	145.25
	0.15	134.86	138.30	137.40	131.39	128.40
	0.20	113.74	113.06	112.01	109.82	109.73
	0.25	94.09	96.75	97.09	93.68	91.82
	0.30	78.22	80.65	77.32	77.27	75.01
	0.50	37.42	40.00	39.81	34.02	34.51
EQL		5.17	5.38	5.29	4.95	4.90
0.25	0.00	199.78	161.05	162.75	138.43	136.26
	0.05	177.58	140.81	141.28	120.26	117.83
	0.10	152.78	122.76	124.99	108.18	105.76
	0.15	130.11	102.13	102.85	89.48	86.21
	0.20	112.72	88.76	84.69	73.04	70.07
	0.25	94.16	72.13	71.77	61.59	57.48
	0.30	78.28	58.57	57.86	47.09	42.90
	0.50	37.22	30.04	29.63	25.68	22.97
EQL		5.15	4.01	3.96	3.36	3.08
0.50	0.00	196.02	136.49	139.69	120.73	124.90
	0.05	173.66	117.36	123.36	106.68	107.93
	0.10	149.10	102.97	101.05	93.66	94.96
	0.15	128.51	87.31	86.84	80.45	79.73
	0.20	105.35	72.01	70.80	65.20	64.33
	0.25	88.35	59.82	57.73	53.08	51.17
	0.30	74.69	40.93	41.30	37.85	33.95
	0.50	32.25	20.99	21.79	17.83	16.56
EQL		4.74	2.96	2.99	2.63	2.47
0.75	0.00	198.23	112.24	110.24	95.55	94.19
	0.05	168.05	100.55	97.09	82.48	80.94
	0.10	150.31	88.61	85.97	70.52	67.90
	0.15	129.11	74.94	73.98	54.55	53.97
	0.20	106.24	60.69	59.04	42.37	39.76
	0.25	93.89	45.50	43.26	30.28	27.60
	0.30	75.92	29.81	27.71	17.09	15.42
	0.50	39.45	17.16	16.56	10.25	9.34
EQL		5.17	2.33	2.23	1.46	1.35
1.00	0.00	197.31	98.00	96.22	84.41	82.85
	0.05	179.76	83.99	82.79	70.42	68.89
	0.10	155.45	67.94	66.65	64.56	59.14

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
	0.15	127.36	55.39	54.76	54.59	50.18
	0.20	100.80	41.00	40.32	40.52	36.03
1.00	0.25	87.84	27.48	26.65	25.38	25.89
	0.30	69.99	19.11	17.49	16.16	13.54
	0.50	36.11	12.77	10.52	8.68	7.90
EQL		4.82	1.61	1.45	1.32	1.19
UCL		7.27	14.36	14.17	7.27	7.27

The performances of weakly auto-correlated *(ρ =0.20)* are given in *Tables (5.13 - 5.20)* above, the parametric charts in all cases had the better performances compared to the non-parametric charts when there is no model misspecification *(γ=0.0)*, but in fact, the semi-parametric charts had superior performances, except when *m=600* and *n=10* of both MEWMA and MCUSUM charts where their performances were exactly the same as the parametric charts. Furthermore, increasing the misspecification amount in the model results in worse efficiencies for the parametric charts but better efficiency for the non-parametric charts. Of course, in all situations, the semi-parametric charts had the best abilities in detecting different amounts of shifts among all level of misspecifications, even though as the misspecification level increases, the performances of the semi-parametric (MMRPM and MMRRPM) charts get closer to the non-parametric charts, yet they are better based on both ARL and EQL results. The results of ATS and SDRL in the Appendix agree with ARL results. Furthermore, we have plotted ARL graphs for some results (*Figure 5.4)*, we can see that the non-parametric charts performed better than the parametric charts as the misspecification increases, also, we can see that the proposed semi-parametric MEWMA & MCUSUM charts were the best in all the plotted cases.

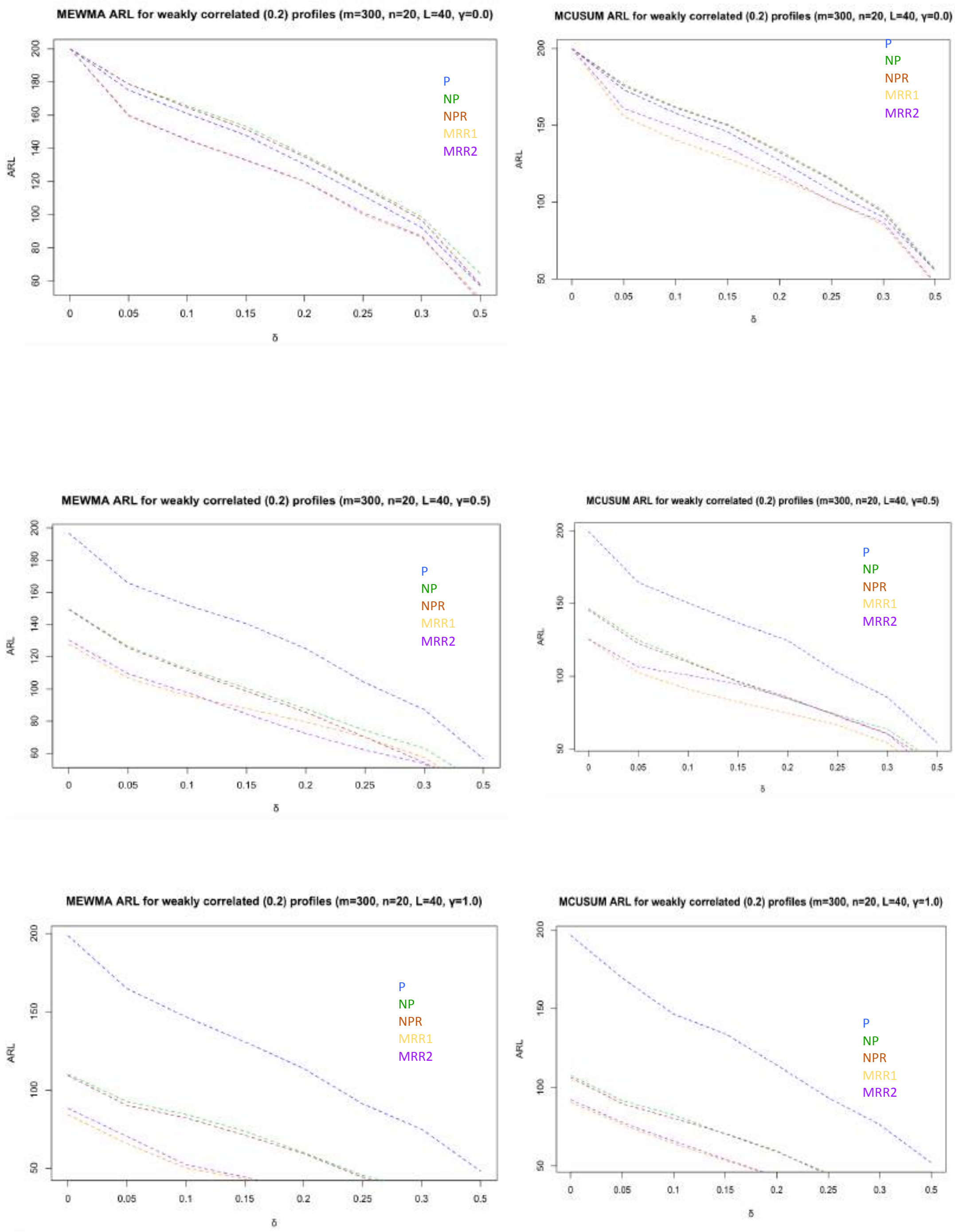

Figure 5.4. ARL graphs for weakly correlated (ρ =0.2) profiles

Table 5.21. MEWMA ARL & EQL for Moderate Auto-Correlated (ρ =0.5) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=10 , l=40)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	171.85	178.46	179.94	171.85	171.85
	0.10	150.85	155.63	154.78	150.85	150.85
	0.15	132.94	135.13	133.97	132.94	132.94
	0.20	117.23	123.73	120.24	117.23	117.23
	0.25	104.34	108.26	107.89	104.34	104.34
	0.30	90.65	94.56	95.96	90.65	90.65
	0.50	53.27	55.96	57.05	53.27	53.27
EQL		6.31	6.60	6.66	6.31	6.31
0.25	0.00	198.66	165.41	168.85	147.17	148.14
	0.05	169.85	142.31	140.15	132.21	130.42
	0.10	147.98	131.32	130.89	114.99	111.13
	0.15	130.16	112.01	110.73	104.32	100.11
	0.20	112.52	97.89	97.57	95.50	94.20
	0.25	100.64	84.88	83.61	83.71	80.13
	0.30	85.41	69.83	70.11	70.72	68.24
	0.50	51.14	40.40	39.09	35.55	33.23
EQL		6.04	4.93	4.85	4.65	4.44
0.50	0.00	195.74	142.73	142.29	132.42	133.47
	0.05	168.24	111.28	107.32	120.92	118.13
	0.10	142.70	103.44	99.74	103.94	100.94
	0.15	127.90	90.21	86.96	91.93	87.31
	0.20	109.32	81.54	76.92	78.38	75.52
	0.25	95.35	73.67	70.86	65.15	63.93
	0.30	78.65	66.09	61.16	53.49	51.10
	0.50	49.75	37.53	33.91	28.62	26.00
EQL		5.76	4.48	4.14	3.69	3.47
0.75	0.00	198.27	128.23	130.62	110.14	111.26
	0.05	164.76	105.85	108.33	98.18	95.28
	0.10	144.33	95.68	94.22	86.13	82.02
	0.15	123.36	71.67	70.12	69.90	70.24
	0.20	105.49	61.37	58.84	59.58	56.42
	0.25	93.78	49.73	45.56	45.47	44.99
	0.30	76.91	39.07	37.10	32.13	32.52
	0.50	49.66	19.67	16.61	13.59	14.63
EQL		5.68	2.70	2.46	2.19	2.23
1.00	0.00	196.64	105.68	104.62	91.38	90.11
	0.05	162.09	90.12	89.35	82.82	78.63
	0.10	143.46	73.78	75.84	70.97	69.46

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
1.00	0.15	125.40	59.38	59.08	55.42	55.42
	0.20	104.53	46.23	45.54	43.75	41.24
	0.25	90.50	33.19	32.05	30.80	26.57
	0.30	77.59	20.12	20.76	17.87	14.68
	0.50	50.46	12.57	10.63	9.31	6.54
EQL		5.71	1.68	1.61	1.45	1.20
UCL		33.02	47.00	47.51	33.02	33.02

Table 5.22. MEWMA ARL & EQL for Moderate Auto-Correlated (ρ =0.5) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=20 , l=40)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	166.41	171.44	170.68	153.50	166.41
	0.10	143.57	140.47	144.79	140.66	143.57
	0.15	130.73	129.53	129.79	125.20	130.73
	0.20	110.51	108.69	109.79	107.98	110.51
	0.25	91.11	94.07	97.07	84.55	91.11
	0.30	75.58	79.54	80.67	70.55	75.58
	0.50	46.38	53.00	50.06	40.35	44.38
EQL		5.51	5.93	5.84	5.02	5.51
0.25	0.00	197.10	167.48	171.20	160.74	163.26
	0.05	165.34	155.99	158.36	144.53	149.93
	0.10	140.58	139.96	139.77	120.70	124.77
	0.15	125.70	122.35	120.51	94.31	100.22
	0.20	104.01	98.18	94.56	81.57	86.11
	0.25	90.10	75.71	68.41	68.36	70.70
	0.30	72.87	57.40	54.97	50.43	52.51
	0.50	44.43	34.78	35.49	27.23	29.81
EQL		5.30	4.35	4.26	3.61	3.84
0.50	0.00	197.09	156.62	159.68	141.28	146.83
	0.05	163.22	137.33	137.09	122.57	125.09
	0.10	142.67	115.83	113.57	106.78	110.73
	0.15	120.46	96.33	95.09	83.81	91.61
	0.20	100.81	70.02	72.88	63.15	72.09
	0.25	86.14	57.61	56.84	50.08	56.87
	0.30	73.79	44.37	42.34	37.75	40.25
	0.50	44.50	25.83	24.60	20.55	21.85
EQL		5.28	3.29	3.19	2.76	2.99

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.75	0.00	198.71	140.20	145.30	127.68	130.95
	0.05	158.25	127.59	131.58	112.71	117.95
	0.10	140.74	100.36	106.77	97.95	102.21
	0.15	122.35	87.47	90.88	83.81	80.20
	0.20	100.40	73.25	69.43	70.60	65.06
	0.25	82.34	52.96	52.07	50.48	49.85
	0.30	69.21	38.26	40.76	32.02	39.67
	0.50	40.54	20.41	21.71	15.65	21.07
EQL		4.95	2.83	2.94	2.41	2.83
1.00	0.00	197.19	136.05	137.09	110.16	114.31
	0.05	161.91	115.89	120.20	94.18	90.54
	0.10	140.87	97.69	96.25	79.29	78.55
	0.15	118.34	85.28	83.73	59.24	64.45
	0.20	100.58	70.99	64.71	43.27	47.41
	0.25	83.51	34.68	40.71	29.26	33.12
	0.30	70.16	20.46	23.36	15.92	17.82
	0.50	41.67	11.52	12.61	6.55	6.73
EQL		5.03	1.85	1.98	1.27	1.38
UCL		32.74	46.31	46.59	32.74	32.74

Table 5.23. MEWMA ARL & EQL for Moderate Auto-Correlated (ρ =0.5) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=10 , l=70)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	166.39	183.70	182.73	166.39	166.39
	0.10	151.22	165.96	164.98	151.22	151.22
	0.15	129.73	146.21	144.94	129.73	129.73
	0.20	108.46	123.09	123.42	108.46	108.46
	0.25	90.29	105.89	102.25	90.29	90.29
	0.30	71.59	88.37	86.85	71.59	71.59
	0.50	39.82	53.71	50.26	39.82	39.82
EQL		5.08	6.36	6.13	5.08	5.08
0.25	0.00	199.94	164.35	162.05	148.35	146.84
	0.05	172.69	145.59	143.46	132.38	129.39
	0.10	154.16	122.24	117.37	115.68	111.22
	0.15	136.17	105.81	103.85	97.11	96.73
	0.20	118.67	86.84	85.70	82.02	80.46
	0.25	96.48	70.22	71.64	65.28	64.29

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.25	0.30	81.92	58.13	56.71	50.48	48.59
	0.50	46.63	34.86	33.10	30.13	29.82
EQL		5.75	4.23	4.10	3.74	3.66
0.50	0.00	196.43	144.42	142.30	130.45	127.87
	0.05	170.65	129.37	126.13	116.98	114.33
	0.10	154.22	108.52	106.77	100.92	97.32
	0.15	136.40	95.70	93.41	87.37	85.79
	0.20	114.80	82.39	80.29	69.81	68.21
	0.25	96.47	68.23	65.04	55.64	56.44
	0.30	80.87	55.34	53.80	43.34	42.57
	0.50	40.82	30.70	28.88	25.94	23.91
EQL		5.42	3.89	3.73	3.22	3.09
0.75	0.00	199.77	125.14	124.66	109.14	105.08
	0.05	174.15	113.11	111.03	96.15	92.94
	0.10	148.08	95.64	94.35	83.92	80.59
	0.15	133.10	81.92	79.23	70.76	67.36
	0.20	108.98	69.48	67.04	58.42	54.17
	0.25	92.52	56.17	55.09	45.30	41.14
	0.30	78.26	43.22	42.59	33.14	29.06
	0.50	36.89	22.11	22.17	17.01	15.39
EQL		5.11	3.01	2.97	2.38	2.15
1.00	0.00	195.04	98.22	98.46	85.26	83.82
	0.05	169.79	87.23	86.85	74.20	73.75
	0.10	144.10	75.16	73.33	59.00	56.96
	0.15	119.14	58.44	56.34	48.71	45.92
	0.20	100.15	45.70	44.54	39.42	34.81
	0.25	86.51	36.93	33.59	28.11	26.78
	0.30	69.02	23.16	21.61	16.42	13.72
	0.50	34.28	14.17	13.32	7.31	6.26
EQL		4.66	1.87	1.76	1.25	1.11
UCL		31.73	44.94	45.28	31.73	31.73

Table 5.24. MEWMA ARL & EQL for Moderate Auto-Correlated (ρ =0.5) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=20 , l=70)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	175.35	179.89	181.91	165.37	168.39
	0.10	139.12	155.91	157.33	132.48	136.52
	0.15	118.11	134.23	134.98	114.76	116.58
	0.20	99.35	112.61	114.53	92.95	94.35
	0.25	85.30	97.68	96.53	80.01	79.30
	0.30	62.84	75.40	74.03	58.41	56.84
	0.50	33.02	42.35	41.92	30.03	29.02
EQL		4.44	5.37	5.32	4.11	4.04
0.25	0.00	199.57	169.36	168.36	145.20	143.24
	0.05	167.39	150.29	150.11	124.42	119.56
	0.10	150.69	136.11	135.75	106.29	104.23
	0.15	134.64	119.94	116.63	99.55	90.29
	0.20	110.58	100.26	98.39	85.39	78.39
	0.25	90.36	83.15	82.16	63.66	60.22
	0.30	75.49	67.25	64.40	44.39	41.69
	0.50	35.73	30.25	31.39	23.28	21.49
	0.50	35.73	30.25	31.39	23.28	21.49
EQL		4.98	4.38	4.36	3.26	3.03
0.50	0.00	197.40	154.27	153.39	136.63	132.67
	0.05	158.32	133.76	132.39	114.63	112.63
	0.10	144.62	118.37	117.63	98.37	96.58
	0.15	127.15	99.55	97.31	87.45	83.63
	0.20	105.49	80.43	83.26	72.48	67.11
	0.25	88.57	66.21	68.42	55.78	53.68
	0.30	69.72	48.58	50.65	37.38	35.43
	0.50	34.69	27.37	27.09	20.58	18.53
EQL		4.74	3.57	3.62	2.83	2.64
0.75	0.00	196.77	137.52	136.42	119.48	116.21
	0.05	169.35	120.48	119.52	104.57	99.38
	0.10	145.62	100.58	98.37	90.32	84.67
	0.15	130.58	85.63	83.52	78.42	72.56
	0.20	108.57	70.44	68.47	63.46	58.63
	0.25	94.82	56.42	53.57	50.28	45.23
	0.30	75.84	43.27	40.28	33.21	30.59
	0.50	37.48	24.21	21.78	17.48	16.52
EQL		5.08	3.14	2.92	2.48	2.30
1.00	0.00	199.48	116.73	115.72	100.47	96.47
	0.05	166.73	100.38	99.26	86.42	84.23
	0.10	143.79	84.67	82.63	70.23	68.31

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
	0.15	127.58	70.27	67.35	57.36	55.32
	0.20	105.62	57.38	55.69	44.62	40.28
1.00	0.25	89.47	40.27	39.86	30.28	27.47
	0.30	74.62	27.48	26.52	19.36	17.38
	0.50	36.73	18.37	15.83	10.37	8.35
EQL		4.96	2.28	2.11	1.54	1.35
UCL		31.16	45.63	45.06	31.16	31.16

Table 5.25. MCUSUM ARL & EQL for Moderate Auto-Correlated (ρ =0.5) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=10 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	169.68	171.87	171.28	169.68	169.68
	0.10	156.29	155.88	154.03	156.29	156.29
0.00	0.15	130.30	138.25	135.27	130.30	130.30
	0.20	113.19	120.64	119.25	113.19	113.19
	0.25	100.21	103.62	101.23	100.21	100.21
	0.30	86.63	91.56	89.8	86.63	86.63
	0.50	50.44	56.36	54.55	50.44	50.44
EQL		6.04	6.51	6.35	6.04	6.04
	0.00	199.20	158.84	160.21	149.96	140.29
	0.05	165.45	140.06	140.70	134.94	120.83
	0.10	152.78	121.74	122.05	120.96	109.72
0.25	0.15	136.43	109.27	106.16	109.76	95.63
	0.20	114.35	95.56	96.01	95.44	88.42
	0.25	99.42	85.43	89.88	80.96	78.11
	0.30	84.64	75.69	77.43	66.57	70.39
	0.50	49.74	46.79	48.25	33.09	35.72
EQL		5.97	5.36	5.49	4.44	4.56
	0.00	198.73	139.41	138.44	125.73	127.51
	0.05	170.38	115.25	115.72	110.59	110.36
	0.10	150.33	102.37	100.94	97.53	97.04
0.50	0.15	136.09	90.76	89.83	84.40	84.36
	0.20	113.12	75.14	77.78	70.19	72.32
	0.25	98.31	61.58	64.27	53.10	55.71
	0.30	86.51	54.43	52.93	40.87	38.55
	0.50	50.12	32.83	31.63	22.84	20.27

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
EQL		6.01	3.88	3.81	2.98	2.83
0.75	0.00	199.96	118.94	118.29	112.65	114.21
	0.05	168.80	100.53	99.28	88.43	94.12
	0.10	149.43	88.19	86.42	74.26	82.16
	0.15	133.17	69.16	67.09	62.37	67.16
	0.20	115.27	55.71	53.34	50.18	50.18
	0.25	98.96	43.52	42.81	38.20	38.20
	0.30	85.95	32.22	30.86	27.04	28.04
	0.50	48.34	17.21	15.03	14.57	15.15
EQL		5.92	2.34	2.18	2.01	2.10
1.00	0.00	198.01	113.80	110.47	99.84	94.73
	0.05	166.28	100.24	99.33	87.43	82.47
	0.10	137.54	79.79	77.34	75.11	74.57
	0.15	125.04	61.49	60.69	57.03	62.62
	0.20	112.32	47.64	49.63	45.21	44.07
	0.25	96.12	36.29	38.55	34.15	35.43
	0.30	82.64	24.60	25.75	22.94	23.68
	0.50	49.56	15.29	12.87	10.88	11.43
EQL		5.84	1.97	1.90	1.67	1.73
UCL		7.58	14.90	14.75	7.58	7.58

Table 5.26. MCUSUM ARL & EQL for Moderate Auto-Correlated (ρ =0.5) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=20 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	162.19	169.06	170.90	159.25	162.19
	0.10	140.53	145.17	146.06	139.06	140.53
	0.15	117.45	121.04	120.11	114.51	117.45
	0.20	99.91	101.71	100.42	92.86	99.91
	0.25	83.89	87.76	86.73	79.73	83.89
	0.30	70.65	75.99	69.51	67.96	70.65
	0.50	40.84	43.36	41.24	39.32	40.84
EQL		5.00	5.29	5.02	4.80	5.00
0.25	0.00	199.63	170.91	172.35	150.54	158.74
	0.05	160.03	152.46	150.72	137.94	140.95
	0.10	137.07	135.40	134.72	120.26	129.89
	0.15	115.36	110.20	106.12	100.51	109.94
	0.20	97.82	95.74	93.74	84.92	86.80

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.25	0.25	86.79	79.99	80.81	63.18	71.76
	0.30	74.53	64.30	61.83	49.39	54.32
	0.50	41.05	36.43	35.29	30.82	31.61
EQL		5.09	4.57	4.44	3.76	4.01
0.50	0.00	196.58	151.91	150.26	139.25	138.21
	0.05	155.72	130.82	127.55	120.92	118.4
	0.10	140.21	109.57	107.25	99.85	95.4
	0.15	125.59	91.70	88.04	85.58	83.14
	0.20	106.09	78.61	74.89	69.60	68.01
	0.25	88.40	63.52	59.24	55.45	52.77
	0.30	72.57	45.57	47.06	38.65	34.98
	0.50	39.66	23.64	20.93	17.63	14.27
EQL		5.05	3.26	3.11	2.69	2.41
0.75	0.00	195.46	139.25	137.77	123.27	126.16
	0.05	157.44	124.16	122.38	100.47	100.25
	0.10	138.46	105.56	103.26	84.45	80.61
	0.15	121.85	89.79	88.78	70.46	65.58
	0.20	100.95	75.57	72.05	53.92	53.32
	0.25	88.47	59.95	55.19	39.70	36.19
	0.30	72.95	42.86	40.59	27.95	20.36
	0.50	40.62	20.60	17.41	13.82	11.34
EQL		5.08	2.98	2.75	2.05	1.71
1.00	0.00	196.47	123.18	122.24	106.83	103.39
	0.05	161.33	101.87	103.91	93.00	87.57
	0.10	143.82	79.96	75.61	70.20	65.52
	0.15	120.22	64.04	62.55	58.53	52.68
	0.20	106.07	42.39	44.93	40.35	35.61
	0.25	90.43	30.85	32.40	28.24	27.39
	0.30	75.55	15.96	16.26	13.11	15.21
	0.50	38.44	7.47	7.17	5.52	4.50
EQL		5.06	1.34	1.34	1.13	1.08
UCL		7.81	15.09	14.96	7.81	7.81

Table 5.27. MCUSUM ARL & EQL for Moderate Auto-Correlated (ρ =0.5) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=10 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	175.12	179.62	178.20	175.12	175.12
	0.10	159.16	164.48	162.96	159.16	159.16
	0.15	136.36	143.22	142.12	136.36	136.36
	0.20	112.01	125.53	127.71	112.01	112.01
	0.25	96.07	106.85	108.37	96.07	96.07
	0.30	72.34	83.93	86.14	72.34	72.34
	0.50	43.80	47.54	48.29	43.80	43.80
EQL		5.37	5.96	6.06	5.37	5.37
0.25	0.00	197.38	158.83	159.72	145.29	147.45
	0.05	170.32	142.89	139.03	128.24	131.42
	0.10	156.17	126.21	124.27	113.16	115.31
	0.15	132.37	107.90	109.09	100.36	99.35
	0.20	117.97	93.35	94.90	87.01	85.01
	0.25	100.25	80.53	79.75	70.07	69.87
	0.30	83.63	66.16	67.42	54.34	56.66
	0.50	40.76	37.87	35.02	30.80	31.57
EQL		5.51	4.66	4.55	3.91	4.00
0.50	0.00	197.02	137.88	135.05	123.88	119.23
	0.05	172.56	123.72	121.79	105.72	103.37
	0.10	155.43	106.58	107.04	88.10	86.14
	0.15	130.00	91.29	92.56	76.04	73.57
	0.20	116.70	78.22	77.99	64.97	59.21
	0.25	98.08	64.42	61.98	50.82	47.36
	0.30	80.34	46.43	43.78	34.68	32.57
	0.50	43.48	29.51	26.57	20.81	17.48
EQL		5.55	3.57	3.35	2.68	2.41
0.75	0.00	195.78	116.64	115.08	99.12	95.58
	0.05	170.69	98.62	98.75	85.11	84.54
	0.10	153.39	85.64	84.44	75.18	71.49
	0.15	131.37	73.46	70.01	63.26	62.61
	0.20	107.83	62.44	57.58	54.18	53.55
	0.25	90.22	50.19	45.02	42.12	40.44
	0.30	73.49	36.76	32.29	25.04	23.35
	0.50	38.75	19.03	16.17	14.31	12.62
EQL		5.07	2.61	2.31	1.99	1.85

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
EQL		5.07	2.61	2.31	1.99	1.85
1.00	0.00	198.86	97.30	98.31	84.56	85.59
	0.05	168.79	79.65	79.90	75.77	76.68
	0.10	151.09	67.95	68.23	62.82	63.61
	0.15	130.75	58.71	59.89	50.82	47.98
	0.20	108.12	44.50	43.74	37.81	34.74
	0.25	87.46	35.83	33.95	27.71	22.76
	0.30	66.23	26.43	24.61	15.67	11.59
	0.50	35.10	16.86	14.19	8.99	6.14
EQL		4.71	2.05	1.87	1.32	1.03
UCL		7.00	14.49	14.58	7.00	7.00

Table 5.28. MCUSUM ARL & EQL for Moderate Auto-Correlated (ρ =0.5) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=20 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	168.39	173.27	169.32	154.25	151.20
	0.10	144.52	150.30	144.46	139.53	135.43
	0.15	119.39	129.00	129.39	114.28	110.59
	0.20	100.69	112.59	110.11	97.45	94.24
	0.25	83.25	99.22	97.24	78.41	75.43
	0.30	69.52	77.69	75.39	63.22	60.49
	0.50	35.32	44.29	45.39	30.43	34.98
EQL		4.70	5.51	5.49	4.25	4.38
0.25	0.00	199.45	170.35	169.35	154.63	151.37
	0.05	169.28	148.39	144.25	139.25	135.24
	0.10	153.29	134.28	130.29	113.69	110.43
	0.15	134.42	112.66	111.84	95.39	89.52
	0.20	116.79	94.96	95.52	80.62	74.87
	0.25	90.69	80.39	79.36	67.24	60.56
	0.30	74.92	58.42	57.93	50.11	44.22
	0.50	35.83	31.80	32.74	27.29	22.65
EQL		5.00	4.21	4.23	3.59	3.15
0.50	0.00	195.75	146.32	143.64	130.68	127.52

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.50	0.05	165.34	131.36	129.52	117.42	113.68
	0.10	152.59	116.74	115.63	103.75	97.44
	0.15	133.56	100.48	99.57	88.35	84.37
	0.20	120.38	85.68	83.53	73.47	70.38
	0.25	99.52	68.62	66.74	57.63	54.62
	0.30	71.37	54.68	53.57	44.24	38.87
	0.50	33.62	26.53	24.62	23.62	19.48
EQL		4.88	3.70	3.55	3.16	2.78
0.75	0.00	198.47	131.36	132.56	115.23	114.26
	0.05	167.53	115.24	113.64	102.15	100.39
	0.10	148.59	100.64	99.48	89.32	85.63
	0.15	129.63	87.35	85.63	70.52	66.51
	0.20	105.63	70.48	68.39	58.63	52.36
	0.25	93.46	54.21	52.36	41.46	40.29
	0.30	76.72	39.51	38.64	28.62	26.41
	0.50	34.62	21.64	20.48	17.41	15.62
EQL		4.94	2.91	2.81	2.28	2.09
1.00	0.00	199.48	117.48	116.38	99.35	95.34
	0.05	166.42	102.46	100.37	85.26	80.37
	0.10	145.31	90.18	87.47	70.42	67.31
	0.15	130.58	76.32	73.56	57.68	53.46
	0.20	109.47	60.38	57.38	45.42	40.55
	0.25	95.63	47.56	45.21	28.46	25.42
	0.30	79.37	30.48	27.48	18.46	14.67
	0.50	36.52	16.52	14.52	9.37	6.42
EQL		5.12	2.33	2.13	1.46	1.17
UCL		7.25	14.75	14.44	7.25	7.25

Tables (5.21 – 5.28) showed the results for MEWMA and MCUSUM charts for moderate *(ρ=0.5)* auto-correlated profiles, when *n=10* with varying profiles *m=300* and *m=600,* both the semi-parametric MEWMA and MCUSUM charts performed as the parametric charts when there is no misspecification in the model *(γ =0.0).* Furthermore, when *n=20* and *m=300*, MMRRPM charts gave the same results as the parametric chart and this is because the mixing parameter (given in chapter 3) is zero, and in this case, MMRPM performed better than MMRRPM as it has lower ARL_1s.

The out-of-control average run length ARL_1 of the non-parametric charts becomes less compared to the parametric and the model misspecification increases, but not better than the semi-parametric techniques. We can see that the performances of both non-parametric NP and NPR charts are close to each other. Also, both the semi-parametric performances MMRPM and MMRRPM are similar to each other, even though one of them can be slightly better than the other, yet they are close. We can see that the decrease in ARL_1 values becomes slower and less as misspecification increases for non-parametric and semi-parametric charts. Moreover, the EQL results are the best for the proposed semi-parametric charts compared to the parametric and the non-parametric charts, this is the same conclusion we got by using SDRL and ATS criteria (attached in the Appendix). In *Figure 5.5*, we have plotted the ARL graphs for *m=600* and *n=10*, as it can be seen from the graphs, when there is no model misspecification, the parametric and the semi-parametric charts performed the same and better than the non-parametric charts, but, when the misspecification increases, the parametric performances decrease and the proposed charts performances increases, thus better in detecting out of control profiles.

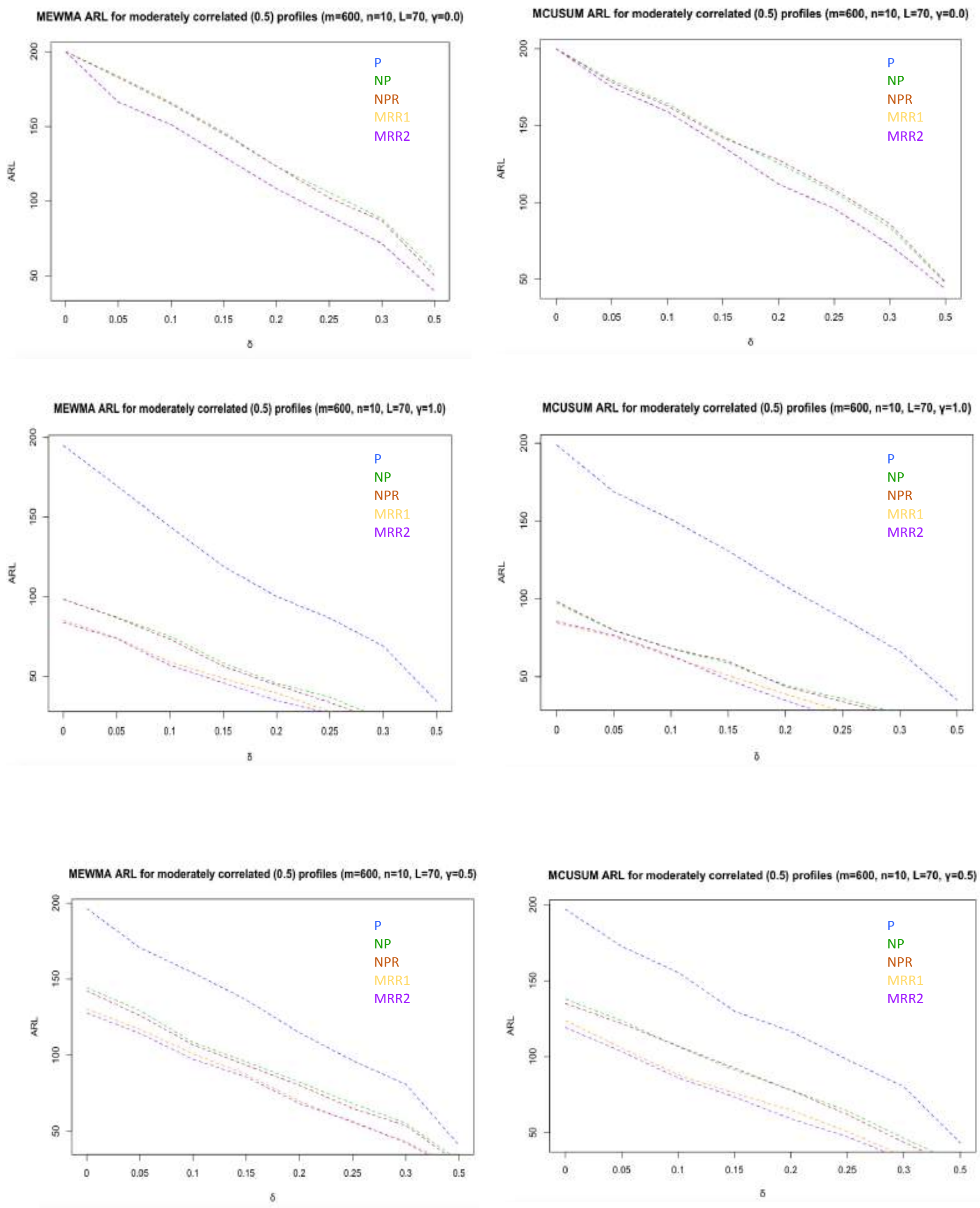

Figure 5.5. ARL graphs for moderately correlated (ρ =0.5) profiles

Table 5.29. MEWMA ARL & EQL for Strong Auto-Correlated (ρ =0.8) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=10 , l=40)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	172.60	180.46	181.47	150.39	147.60
	0.10	155.10	161.49	161.74	132.76	129.10
	0.15	140.09	145.06	146.36	117.22	107.04
	0.20	127.38	130.80	133.00	102.00	94.38
	0.25	105.06	109.08	110.76	90.62	81.06
	0.30	82.05	94.01	93.88	74.11	65.05
	0.50	43.67	55.78	51.30	40.35	36.67
EQL		5.71	6.64	6.43	5.09	4.58
0.25	0.00	196.27	159.89	152.17	137.86	135.01
	0.05	176.70	140.60	136.18	124.33	124.84
	0.10	153.10	123.94	123.90	110.99	106.54
	0.15	139.87	109.83	108.92	94.78	91.61
	0.20	122.38	98.92	99.24	80.95	79.97
	0.25	103.06	81.23	80.60	68.90	67.24
	0.30	84.05	69.88	65.66	56.40	55.12
	0.50	40.90	34.65	33.01	25.39	22.37
EQL		5.58	4.61	4.43	3.64	3.24
0.50	0.00	196.41	134.91	137.86	121.83	125.94
	0.05	177.17	119.86	121.90	101.91	107.99
	0.10	145.90	100.06	102.73	91.80	94.06
	0.15	128.25	89.52	88.52	78.75	79.89
	0.20	109.86	78.50	75.52	65.28	65.74
	0.25	92.92	65.74	62.72	50.03	53.78
	0.30	80.97	50.50	49.50	36.60	35.62
	0.50	44.58	26.46	28.47	18.69	18.19
EQL		5.54	3.51	3.56	2.62	2.61
0.75	0.00	198.17	122.31	120.03	111.40	106.38
	0.05	173.81	105.22	106.99	99.54	93.44
	0.10	140.32	90.26	89.82	83.18	80.08
	0.15	124.00	77.42	77.53	70.00	65.90
	0.20	107.98	63.41	62.16	58.67	51.52
	0.25	90.86	52.39	50.65	49.58	37.58
	0.30	76.23	39.38	37.77	30.18	25.18
	0.50	43.46	21.52	20.90	17.74	13.95
EQL		5.35	2.83	2.75	2.37	1.95
1.00	0.00	195.35	104.29	110.81	99.57	95.37
	0.05	170.44	92.89	96.40	85.83	82.63

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
1.00	0.10	144.88	85.23	87.64	74.07	69.83
	0.15	129.32	67.19	65.23	62.94	58.58
	0.20	104.39	54.38	52.92	49.45	44.81
	0.25	90.13	42.93	40.86	35.49	30.20
	0.30	73.45	28.87	28.43	21.38	17.89
	0.50	42.90	16.29	17.09	10.97	8.91
EQL		5.25	2.20	2.21	1.68	1.43
UCL		32.85	467.50	47.52	32.85	32.85

Table 5.30. MEWMA ARL & EQL for Strong Auto-Correlated (ρ =0.8) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=20 , l=40)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	167.97	175.79	173.81	163.20	167.97
	0.10	156.77	159.80	156.48	153.00	156.77
	0.15	132.65	141.36	140.59	130.17	132.65
	0.20	118.25	122.03	125.35	115.85	118.25
	0.25	100.00	107.05	106.69	99.80	100.00
	0.30	85.61	94.24	89.09	84.07	85.61
	0.50	51.72	63.07	59.45	49.81	51.72
EQL		6.11	6.95	6.66	5.95	6.11
0.25	0.00	196.16	183.44	183.20	164.99	160.32
	0.05	175.05	158.64	156.92	141.30	139.56
	0.10	161.40	145.38	140.21	127.87	125.75
	0.15	140.91	129.06	125.71	110.77	108.63
	0.20	127.23	115.02	110.29	95.74	93.62
	0.25	102.81	91.74	86.09	79.20	78.79
	0.30	84.51	73.28	69.57	62.42	63.42
	0.50	50.84	39.70	38.27	29.64	25.83
EQL		6.11	5.14	4.92	4.17	3.99
0.50	0.00	199.07	162.95	157.92	142.24	140.74
	0.05	169.52	148.68	145.22	130.44	127.35
	0.10	154.85	132.02	130.17	116.32	110.68
	0.15	140.14	119.66	117.17	100.77	96.85
	0.20	123.06	101.64	100.30	87.14	83.85

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
	0.25	103.28	83.20	79.41	68.66	64.67
0.50	0.30	86.50	69.96	64.94	55.03	50.48
	0.50	49.14	30.48	28.98	24.60	21.56
EQL		6.05	4.46	4.23	3.62	3.31
	0.00	199.58	151.48	151.99	129.73	127.84
	0.05	174.01	130.77	129.05	110.74	110.38
	0.10	149.85	116.20	113.13	99.52	95.74
0.75	0.15	134.32	101.35	96.72	85.64	81.23
	0.20	120.89	89.64	87.75	62.51	59.32
	0.25	102.04	73.14	69.24	46.17	42.50
	0.30	84.07	50.63	48.05	33.77	29.06
	0.50	51.85	26.77	25.45	17.53	14.63
EQL		6.10	3.67	3.50	2.49	2.19
	0.00	198.71	128.94	126.33	108.58	102.49
	0.05	170.06	109.46	107.85	91.52	85.74
	0.10	148.22	95.78	94.45	76.70	73.53
1.00	0.15	128.57	80.84	80.60	64.52	59.65
	0.20	109.47	67.48	65.76	50.16	47.16
	0.25	87.28	55.80	53.72	36.91	32.68
	0.30	73.49	38.55	37.11	22.65	19.86
	0.50	42.27	20.63	19.35	11.75	10.68
EQL		5.23	2.82	2.70	1.76	1.60
UCL		32.83	46.49	47.52	32.83	32.83

Table 5.31. MEWMA ARL & EQL for Strong Auto-Correlated (ρ =0.8) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=10 , l=70)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	170.17	177.58	178.02	156.86	165.26
	0.10	152.49	160.95	158.91	140.24	147.11
0.00	0.15	138.33	146.26	145.19	125.07	130.36
	0.20	116.48	125.36	128.52	108.25	114.29
	0.25	99.36	110.67	112.28	96.98	99.12
	0.30	81.45	92.71	91.06	73.74	77.25
	0.50	43.24	51.98	53.37	38.52	42.43
EQL		5.58	6.41	6.46	5.08	5.41
0.25	0.00	199.38	164.72	164.18	143.10	142.28

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
	0.05	184.16	151.06	150.71	130.24	127.56
	0.10	167.01	124.26	124.85	117.27	113.65
	0.15	143.52	103.73	100.33	100.56	100.84
0.25	0.20	120.48	90.09	88.09	85.26	87.49
	0.25	105.92	76.13	72.81	71.69	67.18
	0.30	88.47	61.81	59.31	57.17	56.45
	0.50	48.23	36.67	35.57	29.27	30.68
EQL		6.08	4.45	4.30	3.91	3.94
	0.00	199.91	137.49	134.64	120.95	116.25
	0.05	182.10	118.37	120.71	107.59	101.83
	0.10	165.45	100.72	106.36	95.28	88.76
0.50	0.15	132.62	87.81	82.66	82.31	70.90
	0.20	119.81	69.11	70.98	65.06	58.59
	0.25	92.81	55.77	54.47	49.60	45.48
	0.30	77.55	40.70	40.52	36.17	31.95
	0.50	44.36	23.86	24.56	20.07	17.16
EQL		5.53	3.06	3.08	2.69	2.36
	0.00	194.97	115.14	114.44	95.25	94.39
	0.05	178.95	99.82	96.37	84.98	81.99
	0.10	151.89	85.92	79.90	77.95	74.05
0.75	0.15	133.06	69.54	65.74	64.65	58.73
	0.20	114.47	54.82	52.82	49.56	48.50
	0.25	100.11	38.25	37.93	34.32	33.29
	0.30	74.58	30.00	27.99	26.25	22.10
	0.50	40.26	17.07	16.27	15.50	13.94
EQL		5.27	2.25	2.14	2.02	1.82
	0.00	193.54	98.20	98.46	87.22	87.88
	0.05	169.15	87.38	87.50	79.17	76.61
	0.10	140.10	78.00	75.82	70.25	67.54
1.00	0.15	129.73	66.53	65.76	58.87	57.55
	0.20	102.16	54.07	56.28	46.70	45.37
	0.25	90.90	40.59	41.42	32.25	32.26
	0.30	73.55	28.71	29.01	23.03	23.14
	0.50	37.34	14.79	14.34	12.15	13.57
EQL		4.97	2.10	2.09	1.73	1.79
UCL		31.90	45.34	45.24	31.90	31.90

Table 5.32. MEWMA ARL & EQL for Strong Auto-Correlated (ρ =0.8) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=20 , l=70)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	177.00	181.37	180.62	177.00	169.25
	0.10	161.20	167.61	166.21	161.20	155.26
	0.15	145.64	149.61	148.78	145.64	137.78
	0.20	131.27	134.14	135.30	131.27	122.11
	0.25	112.22	119.14	118.08	112.22	100.95
	0.30	85.48	89.14	88.02	85.48	83.26
	0.50	38.82	43.03	42.45	38.82	34.59
EQL		5.62	5.98	5.92	5.62	5.22
0.25	0.00	199.40	160.74	159.52	149.41	147.01
	0.05	175.84	146.18	145.07	134.25	131.84
	0.10	158.59	129.48	129.17	120.14	116.74
	0.15	144.64	113.35	111.15	104.80	100.74
	0.20	124.54	92.03	96.53	88.17	86.25
	0.25	107.96	76.48	76.18	71.17	70.93
	0.30	86.87	60.76	58.28	56.71	53.39
	0.50	40.88	32.83	31.28	29.47	24.83
EQL		5.69	4.27	4.15	3.93	3.60
0.50	0.00	196.82	143.44	141.22	130.27	125.55
	0.05	168.24	130.12	128.90	114.13	111.47
	0.10	150.99	116.79	114.33	100.07	95.36
	0.15	135.29	97.83	95.85	89.73	85.11
	0.20	110.10	82.61	80.18	76.46	72.96
	0.25	94.09	69.43	67.88	59.56	54.95
	0.30	77.09	54.35	53.84	44.38	42.50
	0.50	38.95	27.94	25.87	23.26	20.00
EQL		5.20	3.75	3.61	3.17	2.91
0.75	0.00	194.39	119.29	119.01	98.83	95.00
	0.05	170.80	98.44	98.12	86.70	83.80
	0.10	147.05	84.56	83.86	74.60	71.63
	0.15	132.00	67.28	66.05	63.35	57.38
	0.20	115.38	51.64	51.68	50.28	45.26
	0.25	96.18	38.38	38.17	34.31	33.17
	0.30	78.46	26.84	25.98	20.90	20.10
	0.50	39.00	15.97	14.99	12.44	11.45
EQL		5.26	2.11	2.03	1.74	1.63
1.00	0.00	195.97	93.34	93.17	77.53	73.79
	0.05	176.65	77.84	77.46	70.57	64.00
	0.10	145.97	66.89	66.49	57.55	53.91
	0.15	125.10	54.89	54.12	46.27	44.82

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
1.00	0.20	109.52	42.64	42.33	34.08	32.47
	0.25	93.15	34.85	33.14	28.91	24.44
	0.30	76.31	23.48	22.63	17.98	13.59
	0.50	35.64	13.07	11.99	9.23	7.86
EQL		4.99	1.78	1.69	1.36	1.15
UCL		31.66	45.10	45.03	31.66	31.66

Table 5.33. MCUSUM ARL & EQL for Strong Auto-Correlated (ρ =0.8) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=10 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	171.30	176.50	174.79	148.30	141.57
	0.10	152.46	153.95	152.95	127.46	122.52
	0.15	136.00	135.80	136.18	106.00	101.58
	0.20	120.71	115.54	117.40	90.71	87.42
	0.25	108.72	101.04	102.96	78.72	74.49
	0.30	79.92	87.01	90.68	64.92	59.21
	0.50	42.58	49.97	50.40	32.10	30.37
EQL		5.59	6.05	6.17	4.32	4.05
0.25	0.00	196.60	146.77	146.60	132.42	130.69
	0.05	169.37	129.84	130.67	119.01	115.74
	0.10	153.47	115.47	118.95	106.99	103.30
	0.15	136.85	101.01	100.71	94.78	90.63
	0.20	123.72	90.92	89.96	80.46	79.29
	0.25	105.70	79.99	78.35	69.75	65.50
	0.30	81.91	67.09	65.94	55.30	51.21
	0.50	45.53	31.26	29.25	27.41	25.52
EQL		5.77	4.31	4.17	3.72	3.49
0.50	0.00	199.28	129.99	125.97	117.91	112.69
	0.05	167.03	115.76	110.63	104.22	100.35
	0.10	148.84	93.35	91.13	88.21	84.74
	0.15	124.91	81.75	82.73	75.45	71.84
	0.20	105.23	70.76	70.25	61.88	59.93
	0.25	90.17	58.80	58.11	49.65	46.79
	0.30	76.15	46.67	41.63	37.50	33.70
	0.50	41.79	26.34	23.64	20.17	15.53
EQL		5.25	3.32	3.06	2.69	2.33

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.75	0.00	196.49	116.66	115.52	106.69	106.45
	0.05	172.00	102.66	99.62	98.83	96.81
	0.10	147.23	87.30	85.15	83.77	79.67
	0.15	121.56	76.82	72.64	68.78	67.75
	0.20	105.11	58.11	56.70	57.71	54.77
	0.25	90.63	45.08	43.08	44.61	42.61
	0.30	77.81	33.38	31.37	31.52	28.52
	0.50	44.61	19.46	18.05	15.48	12.48
EQL		5.43	2.52	2.37	2.25	2.01
1.00	0.00	197.15	103.23	102.82	83.78	80.81
	0.05	166.74	90.13	88.11	74.82	70.71
	0.10	137.10	77.74	76.74	66.54	62.70
	0.15	125.51	60.01	63.88	54.48	50.60
	0.20	100.75	47.47	44.44	42.34	39.55
	0.25	89.57	34.19	31.15	28.25	26.44
	0.30	75.20	22.16	20.16	17.10	15.19
0.50	0.50	44.91	14.37	13.36	9.42	7.69
EQL		5.36	1.85	1.73	1.41	1.24
UCL		7.74	14.82	14.80	7.74	7.74

Table 5.34. MCUSUM ARL & EQL for Strong Auto-Correlated (ρ =0.8) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=300 , n=20 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	163.76	170.22	171.71	159.25	163.76
	0.10	148.64	149.46	146.98	146.18	148.64
	0.15	125.95	131.09	128.98	119.04	125.95
	0.20	104.51	110.29	109.56	100.62	104.51
	0.25	86.40	96.02	95.54	83.88	86.40
	0.30	72.35	75.26	74.53	69.49	72.35
	0.50	44.50	47.33	47.02	43.19	44.50
EQL		5.28	5.58	5.54	5.10	5.28
0.25	0.00	199.99	183.38	182.37	160.54	158.46
	0.05	170.65	160.81	155.94	138.92	135.84
	0.10	154.35	149.62	142.89	120.99	117.83
	0.15	141.11	125.83	125.01	104.72	100.93
	0.20	127.97	108.90	110.02	91.11	86.74
	0.25	104.57	94.64	93.51	77.66	74.32

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.25	0.30	86.86	71.67	70.91	63.51	60.20
	0.50	49.32	37.58	36.23	25.32	22.35
EQL		6.10	4.99	4.89	3.93	3.66
0.50	0.00	197.67	159.65	157.08	138.78	135.73
	0.05	170.82	145.95	141.84	123.97	120.64
	0.10	155.38	131.09	127.18	107.79	103.69
	0.15	141.55	115.65	112.03	94.76	87.85
	0.20	127.51	97.24	95.88	80.41	75.78
	0.25	109.72	80.64	78.29	65.66	53.67
	0.30	89.85	62.38	61.58	47.34	35.29
	0.50	50.02	32.45	30.58	21.49	19.78
EQL		6.23	4.34	4.20	3.22	2.75
0.75	0.00	199.64	142.12	142.20	124.01	126.32
	0.05	172.67	125.42	122.42	107.08	110.53
	0.10	153.18	109.75	109.19	94.15	96.10
	0.15	138.38	96.95	95.72	78.97	80.05
	0.20	116.81	80.23	81.29	65.85	62.35
	0.25	100.22	64.27	63.66	47.72	50.23
	0.30	82.96	50.68	47.42	31.61	30.49
	0.50	48.94	24.02	22.13	18.73	15.30
EQL		5.91	3.43	3.25	2.51	2.31
1.00	0.00	198.81	127.92	128.00	105.99	102.58
	0.05	168.94	112.20	113.63	90.98	85.89
	0.10	150.43	100.49	99.97	76.10	71.42
	0.15	130.62	82.72	82.00	60.08	57.73
	0.20	109.52	68.42	66.16	44.14	40.90
	0.25	97.04	55.99	54.86	28.00	27.09
	0.30	83.51	34.74	35.05	15.83	14.64
	0.50	50.34	20.66	19.90	8.99	8.52
EQL		5.97	2.75	2.70	1.39	1.31
UCL		8.06	14.67	14.73	8.06	8.06

Table 5.35. MCUSUM ARL & EQL for Strong Auto-Correlated (ρ =0.8) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=10 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	170.37	175.58	176.31	154.37	159.67

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.10	144.01	156.90	160.03	137.01	140.52
	0.15	123.06	130.43	142.89	114.06	119.60
	0.20	100.23	110.49	120.11	95.23	99.23
	0.25	87.42	94.48	101.22	79.42	85.42
	0.30	69.69	79.78	81.90	60.69	64.69
	0.50	40.70	44.22	42.38	34.70	36.15
EQL		5.01	5.53	5.60	4.41	4.64
0.25	0.00	198.77	163.88	150.15	137.78	135.91
	0.05	178.37	147.35	129.06	123.71	120.78
	0.10	164.03	128.67	112.18	107.92	106.92
	0.15	140.62	104.06	99.72	95.23	93.46
	0.20	116.23	90.62	85.94	83.88	80.46
	0.25	100.71	71.00	69.02	67.28	64.96
	0.30	79.70	54.51	52.94	54.93	51.65
	0.50	46.74	34.49	32.17	30.66	29.19
EQL		5.74	4.15	3.94	3.87	3.69
0.50	0.00	196.97	136.85	135.14	116.14	114.03
	0.05	172.09	119.80	118.64	103.06	99.95
	0.10	159.06	99.19	104.56	89.38	87.62
	0.15	138.90	86.68	91.44	76.39	74.52
	0.20	120.18	72.19	77.03	65.15	63.90
	0.25	105.75	59.75	62.72	51.17	50.23
	0.30	82.10	46.49	42.51	43.93	38.05
	0.50	46.53	28.10	25.45	24.86	21.69
EQL		5.83	3.43	3.26	3.09	2.79
0.75	0.00	197.63	112.17	119.47	95.28	93.55
	0.05	167.95	97.22	95.41	83.26	83.52
	0.10	150.03	81.51	79.53	69.47	70.78
	0.15	132.07	68.37	66.08	56.46	58.61
	0.20	110.11	52.53	50.85	47.25	47.36
	0.25	90.19	36.17	34.97	35.34	34.52
	0.30	72.11	28.36	26.85	25.15	23.37
	0.50	42.42	16.51	18.26	14.34	12.57
EQL		5.23	2.15	2.19	1.91	1.78
1.00	0.00	195.04	96.32	92.15	82.85	80.83
	0.05	175.00	82.15	80.33	70.93	70.86
	0.10	149.55	71.65	70.69	60.12	63.73
	0.15	133.77	57.38	58.58	52.09	51.13
	0.20	120.78	45.43	43.77	41.95	42.84
	0.25	99.59	33.62	34.13	29.93	29.96
	0.30	67.82	25.28	26.92	18.85	20.89
	0.50	38.76	14.17	16.31	10.13	11.40
EQL		5.06	1.89	2.03	1.48	1.59
UCL		7.12	14.77	14.74	7.12	7.12

Table 5.36. MCUSUM ARL & EQL for Strong Auto-Correlated (ρ =0.8) Profile Datasets for Different Model Misspecification Levels (γ) and Shift Sizes (δ) (m=600 , n=20 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	200.00	200.00	200.00	200.00	200.00
	0.05	176.50	182.25	180.59	176.50	168.50
	0.10	154.81	164.85	163.93	154.81	149.81
	0.15	135.05	143.53	140.26	135.05	131.05
	0.20	122.43	126.74	127.55	122.43	115.43
	0.25	101.66	106.98	100.82	101.66	96.66
	0.30	82.15	90.42	87.69	82.15	78.15
	0.50	37.34	45.53	43.63	37.34	32.34
EQL		5.34	6.02	5.81	5.34	4.92
0.25	0.00	196.54	162.83	160.71	137.99	136.11
	0.05	168.74	146.70	145.69	125.88	123.86
	0.10	146.70	130.36	128.77	108.02	106.09
	0.15	125.11	119.16	116.73	89.94	85.96
	0.20	105.85	98.95	99.61	76.64	75.59
	0.25	90.96	81.91	79.88	62.44	60.31
	0.30	80.58	63.74	62.27	46.96	44.85
	0.50	38.28	33.44	32.86	26.03	23.62
EQL		5.18	4.44	4.36	3.39	3.19
0.50	0.00	198.47	137.27	136.25	123.51	120.97
	0.05	169.39	121.96	118.47	110.40	107.93
	0.10	144.14	104.35	103.42	96.65	95.12
	0.15	128.86	86.56	84.43	81.59	78.05
	0.20	106.06	74.45	71.29	67.52	64.82
	0.25	90.60	57.05	55.65	51.13	48.36
	0.30	75.44	40.33	37.38	35.86	34.16
	0.50	40.20	25.81	24.34	19.83	17.71
EQL		5.17	3.18	3.01	2.69	2.51
0.75	0.00	199.34	121.76	120.20	105.10	103.46
	0.05	168.86	107.98	105.42	95.20	91.54
	0.10	151.31	86.46	85.86	82.38	78.92
	0.15	132.44	74.55	76.46	65.33	63.71
	0.20	110.61	61.22	61.06	52.29	49.51
	0.25	89.31	44.76	43.75	35.08	33.27
	0.30	70.39	31.94	30.09	22.88	20.05
	0.50	36.96	18.34	18.93	14.17	12.08
EQL		4.92	2.44	2.45	1.90	1.71
1.00	0.00	192.60	101.73	100.21	84.24	82.81
	0.05	165.89	90.19	89.50	74.35	73.16

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
	0.10	149.25	77.21	76.15	60.13	56.70
	0.15	130.75	69.76	67.89	46.14	42.68
	0.20	110.90	56.48	55.95	34.03	32.44
1.00	0.25	93.72	40.69	40.00	23.97	22.41
	0.30	69.89	26.62	26.16	14.89	12.30
	0.50	35.86	16.87	15.59	7.57	6.72
EQL		4.87	2.17	2.09	1.18	1.05
UCL		7.15	14.63	14.81	7.15	7.15

For strong *(ρ=0.80)* auto-correlated profiles, the performances of MEWMA and MCUSUM charts are given in *Tables (5.29 – 5.*32) and *Tables (5.33 – 5.36),* respectively. When *n=10* and *m=300*, the results of MMRRPM for both MEWMA and MCUSUM are the best as their ARL_1s are the lowest. In all previous cases, when there is no model misspecification, the parametric technique is superior compared to the non-parametric techniques, but the semi-parametric performances are at least the same or better than the parametric performances. The behaviors of the semi-parametric MMRPM and MMRRPM for MCUSUM and MEWMA are competing in terms of their ARL and EQL values, as in some cases, MMRPM is better, while in other cases, the performance of MMRRPM is better. Also, the non-parametric NP and NPR performances are almost the same, yet they are not better than the performances of the semi-parametric charts among all misspecification levels based on ARL and EQL results. Same conclusions and results were obtained using SDRL and ATS criteria (Tables are attached in the Appendix). In *Figure 5.6*, we are representing the ARL results for *m=600* and *n=20*, we can see that when there is no model misspecification, all the charts had almost same performances, but MMRRPM approach had the best performances. Furthermore, the parametric charts had the worst performances when the misspecification amount increased, while the non-parametrics had better results. We can see that the proposed semi-parametric charts had the best performances.

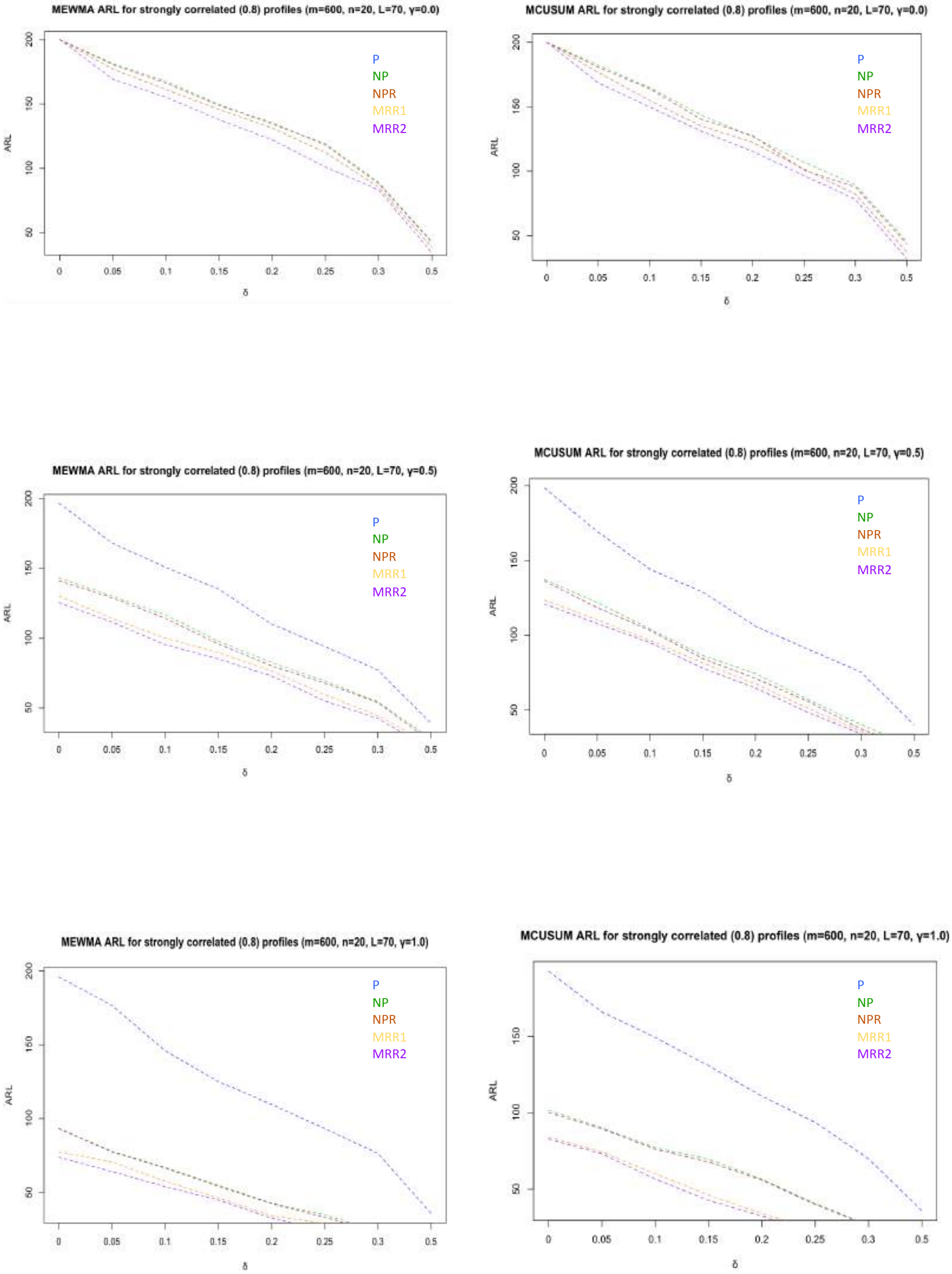

Figure 5.6. ARL graphs for strongly correlated (ρ =0.8) profiles

CHAPTER 6: REAL DATA APPLICATION

In this section, the different control charts (proposed and parametric) discussed early will be illustrated using three real datasets: (1) Apple diameter dataset, (2) Vertical density dataset and (3) Automobile Engine dataset, where they were adapted from the literature.

6.1: Apple Diameter Dataset

This case study was carried out by Schabenberger and Pierce (2002), where a real dataset of ten apple trees were randomly selected and twenty-four apples were randomly chosen from each tree. They concentrated on the apples that had the largest sizes (initial diameters exceeded 2.75 inches) and in total there were eighty apples with that size. The diameters of the apples were recorded eve two weeks over a period of twelve weeks and these measured diameters are shown in *Figure (6.1)*.

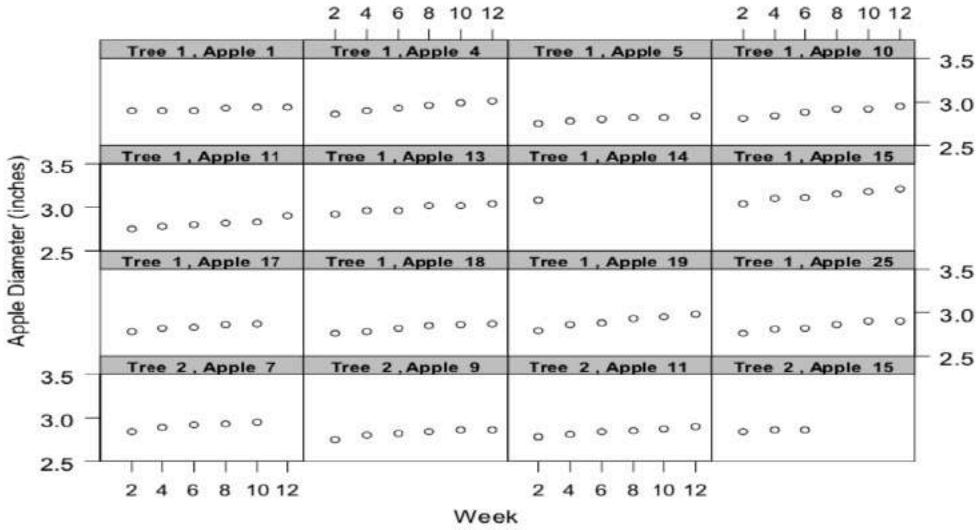

Figure 6.1. Measured diameters of apples during a period of 12 weeks (Schabenberger and Pierce 2002)

The dataset contains the diameters for each of the twenty-four apples recorded for time 1-6. Thus, the data contains 144 diameters for all the apples together. The graph for the raw apple diameter dataset is shown in *Figure (6.2).*

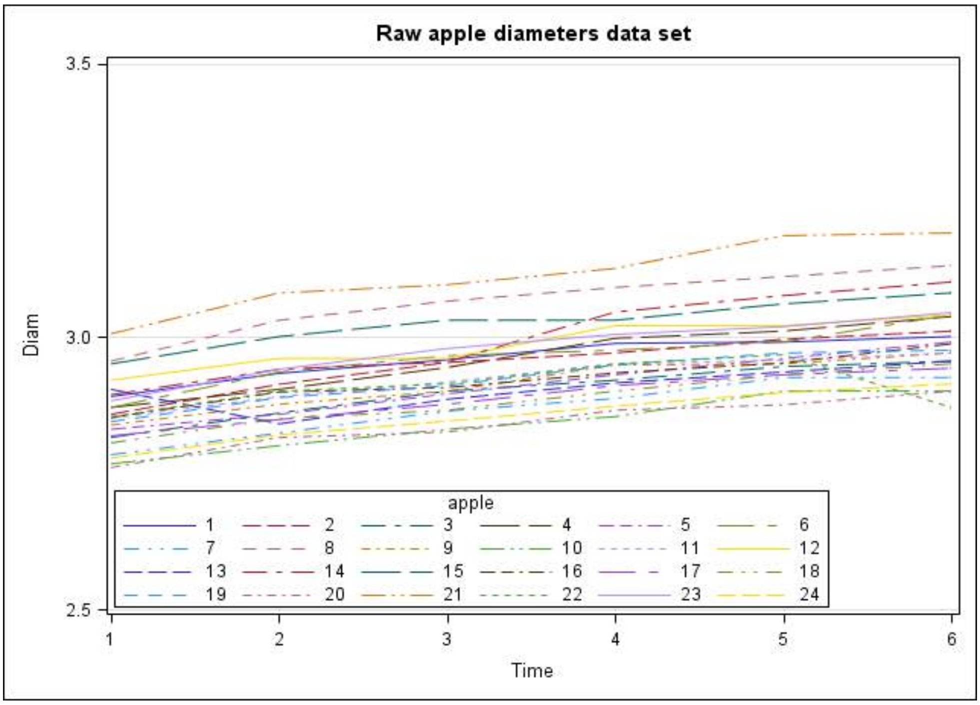

Figure 6.2: Raw data set for apple diameters

The profile between time (explanatory variable) and diameter (response variable) was the quality characteristic that was monitored over time. Furthermore, the correlation between observations was modeled by a first-order autoregressive model AR(1). The simulation scenarios conducted previously for the parametric, non-parametric and semi-parametric techniques will be used for this dataset. Furthermore, since the control limits were obtained through simulation, a linear mixed model *equation (6.1)* was fitted for the data in order to obtain the in-control limits for each control chart:

$$y_{ij} = 2.8406 + b_{0j} + \left(0.02740 + b_{1j}\right)x_i + \varepsilon_{ij} \tag{6.1}$$

$$\varepsilon_{ij} = 0.4429\varepsilon_{i-1j} + a_{ij}$$

where $b_{0j} \sim N(0,0.001214), b_{1j} \sim N(0,0.000017)$ and $a_{ij} \sim N(0,0.1876)$

The normality assumption of residuals to all different techniques was checked by plotting histograms as shown in *Figure (6.3)* for each of them. As it can be seen, the residuals of the parametric estimations are highly skewed to the right which is an indication of a lack of fit using this technique, on the other hand, the histograms of the two non-parametric and the two semi-parametric techniques showed an almost normal distribution. The profiles were fitted using the parametric, non-parametric and semi-parametric techniques mentioned previously and these fits are shown in *Figure (6.4).*

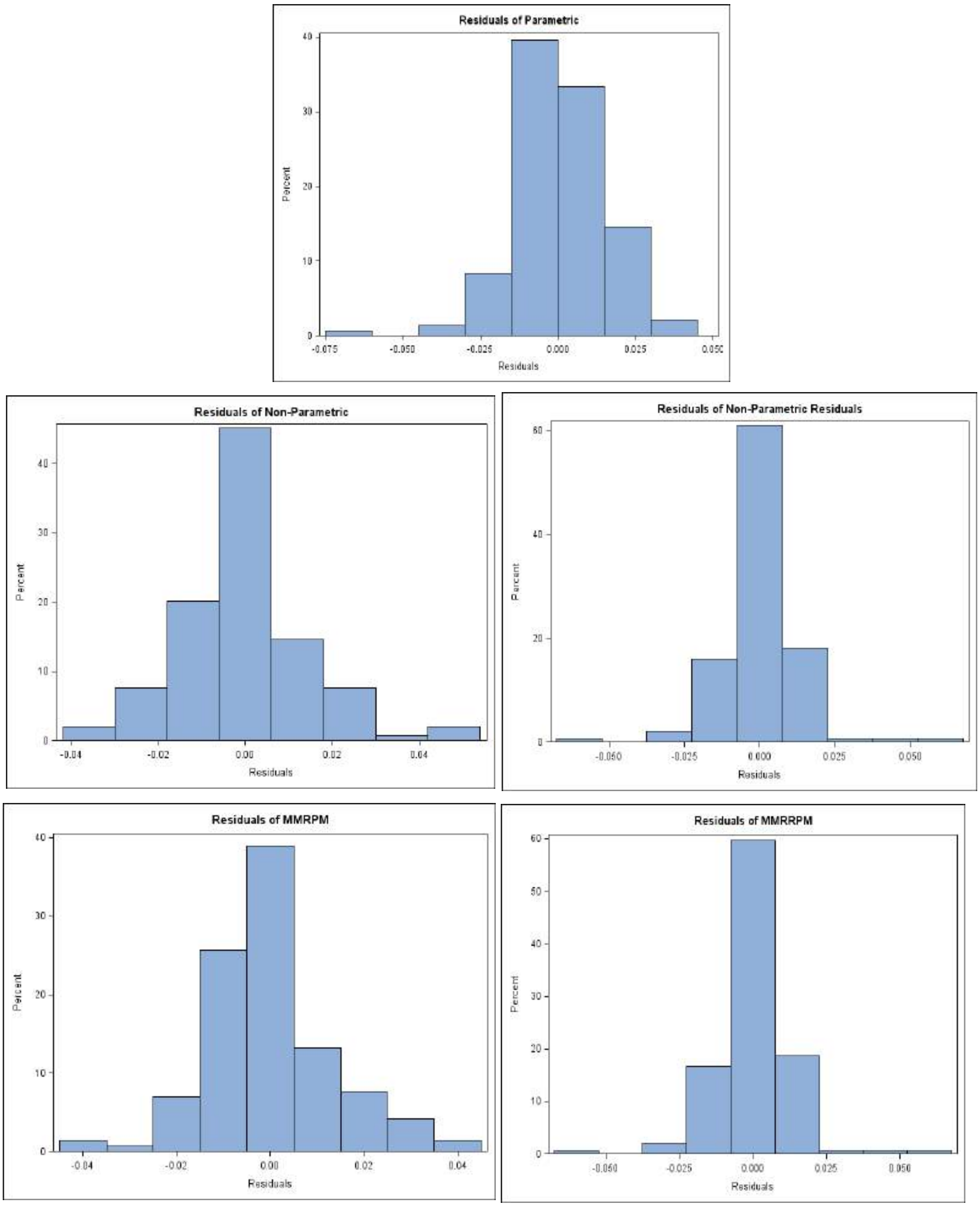

Figure 6.3. Apple Diameter Profile Residuals: (a). Parametric fit, (b). Non-Parametric fit on raw data, (c). Non-Parametric fit on Residuals, (d). Semi-parametric fit MMRPM, (e). Semi-parametric fit MMRRPM

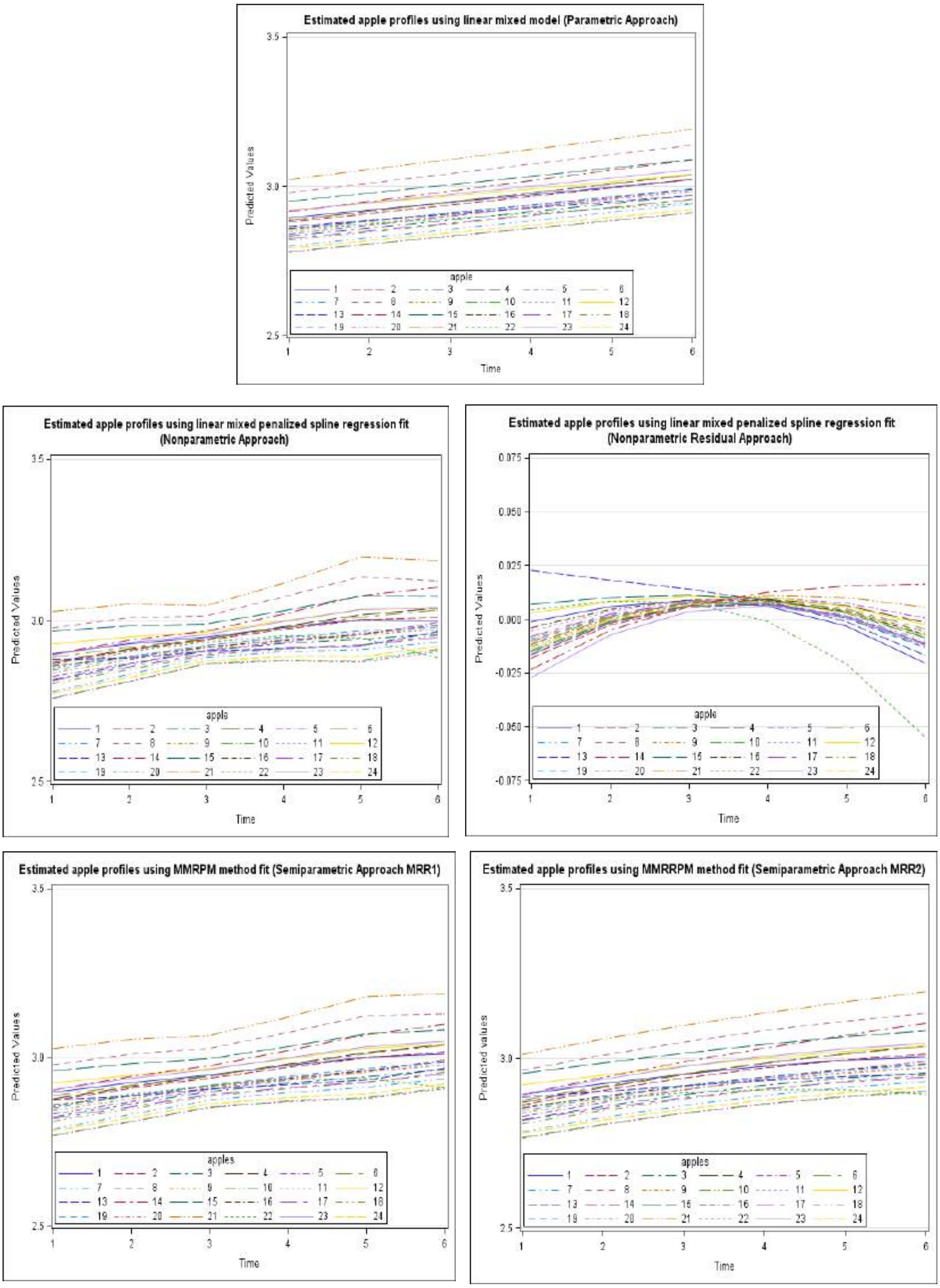

Figure 6.4. Apple Diameter Profiles: (a). Parametric fit, (b). Non-Parametric fit on raw data, (c). Non-Parametric fit on Residuals, (d). Semi-parametric fit MMRPM, (e). Semi-parametric fit MMRRPM

As it can be seen from *Figure (6.4),* the parametric fit method provides linear fits without considering the small bending and thus the parametric method is not capable for modeling this profile data. On the other hand, both non-parametric and MMRPM graphs are close to the raw data graph where they are not presented as linear fits and they consider fluctuations occurring. The MMRRPM technique is almost linear but has little bents and it is close to the parametric fit.

Furthermore, the mean square error MSE for each technique was computed:

- Parametric MSE = 0.000243
- Nonparametric MSE = 0.000209
- Nonparametric residuals MSE = 0.000157
- MMRPM MSE = 0.000167
- MMRRPM MSE = 0.0001561

The parametric fit has the highest MSE compared to other techniques, the non-parametric and the MMRPM techniques have the second and third highest MSE values, respectively, while the non-parametric residuals and MMRRPM techniques have the least MSE.

The upper control limits for the proposed MEWMA and MCUSUM charts for all the five techniques were found to achieve the desired ARL_0=200 based on 10000 simulation runs and for examining the performances of these charts, the first sixteen profiles were considered as in-control, while a slope shift coefficient of 0.5 was inserted to the last eight profiles. The sensitivity of the proposed MEWMA and MCUSUM charts is represented in *Figure (6.5)* and *Figure (6.6),* respectively.

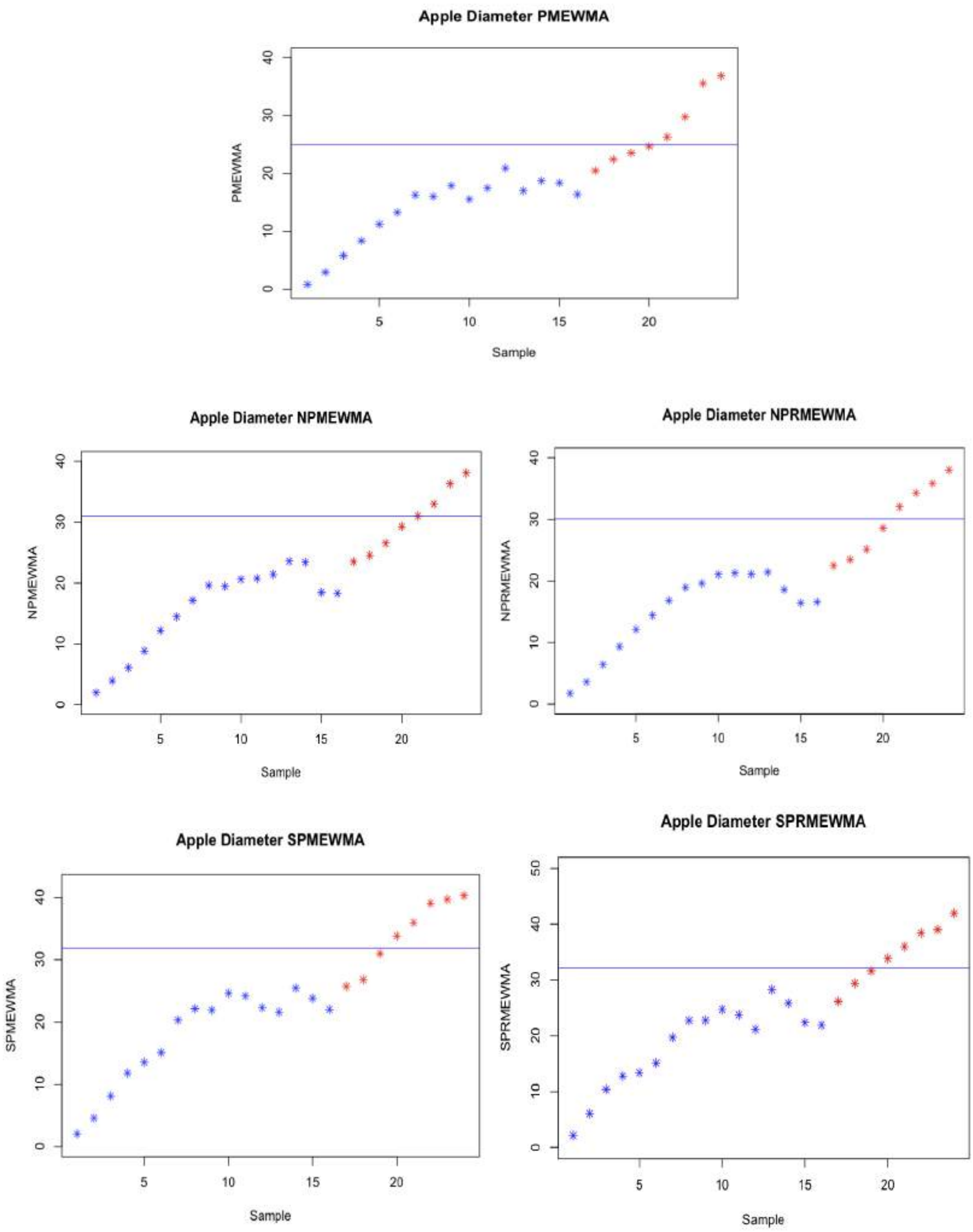

Figure 6.5. Apple Diameter MEWMA: (a). Parametric MEWMA, (b). Non-Parametric MEWMA, (c). Non-Parametric Residuals MEWMA, (d). Semi-parametric MMRPM MEWMA, (e). Semi-parametric MMRRPM MEWMA

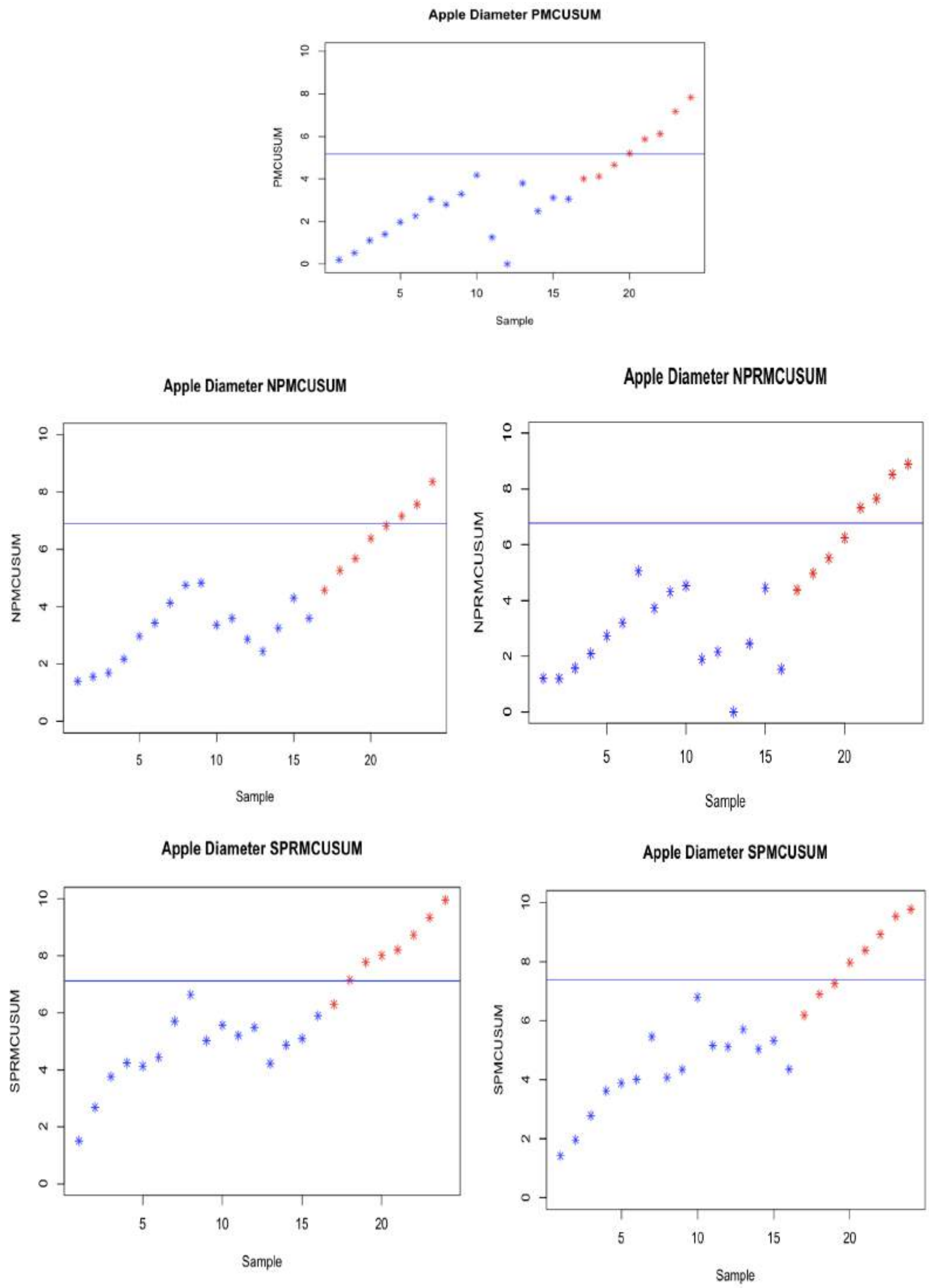

Figure 6.6. Apple Diameter MCUSUM: (a). Parametric MCUSUM, (b). Non-Parametric MCUSUM, (c). Non-Parametric Residuals MCUSUM, (d). Semi-parametric MMRPM MCUSUM, (e). Semi-parametric MMRRPM MCUSUM

Figure (6.5) and *Figure (6.6)* represent the sensitivity of the proposed MEWMA and MCUSUM charts, respectively. The blue points represent the in-control samples, while the red points represent the out-of-control samples where the shift was introduced. From both figures, the semi-parametric charts (SPMEWMA, SPRMEWMA, SPMCUSUM & SPRMCUSUM) are more sensitive in detecting shifts compared to other charts, as fewer profiles are required to detect an out-of-control process, in fact, SPRMCUSUM required only one out-of-control profile for detection. We can see that the performances of the parametric (PMEWMA) and the non-parametric (NPMEWMA & NPRMEWMA) charts are almost the same as almost four profiles are needed to detect an out-of-control process, but we can see that the sensitivity of the parametric (PCUSUM) chart is higher than the sensitivity of the two non-parametric (NPMCUSUM & NPRMCUSUM) charts. We can conclude that since the model is linear, the parametric will perform better than the non-parametric charts, but the semi-parametric charts were better and thus, it is recommended to use these charts for a faster shift detection. For the out of control processes, then the fertilization, water and oxygen rates should be checked to maintain a good growth for the apple.

6.2: Vertical Density Dataset

The vertical density profile (VDP) data was used by Walker and Wright (2002) shown in *Figure (6.7),* where the density for twenty-four wood boards was measured, as it is important for maintaining good quality. Each VDP contains 314 measurements taken 0.002 inches apart.

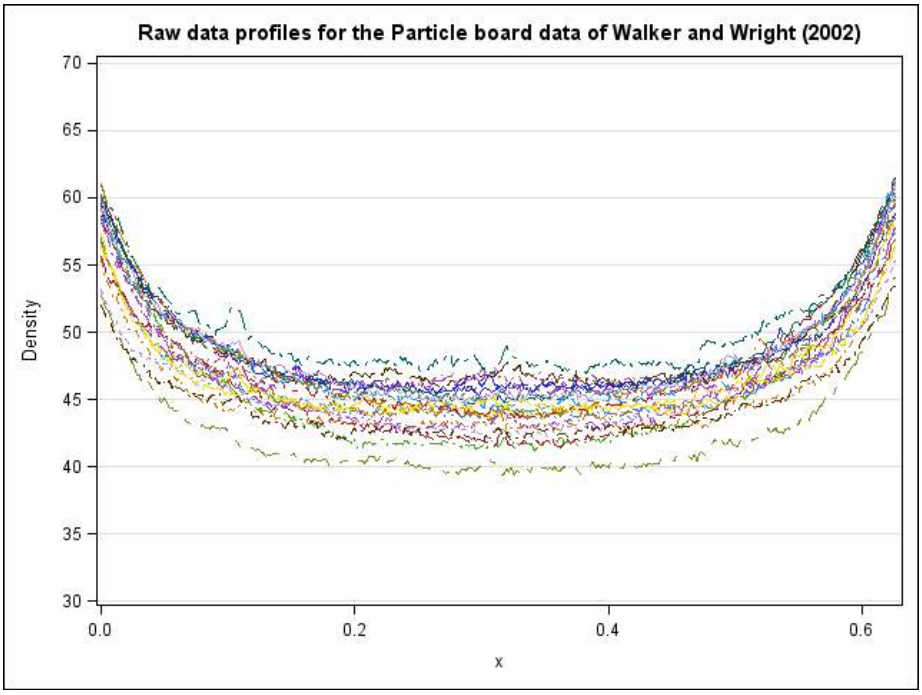

Figure 6.7. Raw dataset for vertical density boards

The response variable (*Y)* is the board density and the explanatory variable *(X)* is the depth measured at interval of 0.002 inches between each two consecutive measures. The profile between the density and the depth was fitted using linear mixed model shown in *equation (6.2)*. The simulation scenarios conducted previously for the parametric, non-parametric and semi-parametric techniques will be used for this dataset, furthermore, since the control limits were obtained through simulation, and the in-control limits for each control chart were computed.

$$y_{ij} = 53.70 + b_{0j} + (-67.55 + b_{1j})x_{ij} + (109.15 + b_{2j})x_{ij}^2 + \varepsilon_{ij} \qquad (6.2)$$

where $b_{0j} \sim N(0,1.62), b_{1j} \sim N(0,46.80)$ and $b_{2j} \sim N(0,121.19)$.

The normality assumption of residuals to all different techniques was checked by plotting histogram as shown in *Figure (6.8)* for each of them. The residuals of the parametric estimations are highly skewed to the left which is an indication of a lack of fit using this technique, on the other hand, the histograms of the two non-parametric and the two semi-parametric techniques showed almost normal distributions. The profiles were fitted using the parametric, non-parametric and semi-parametric techniques mentioned previously and these fits are shown in *Figure (6.9).*

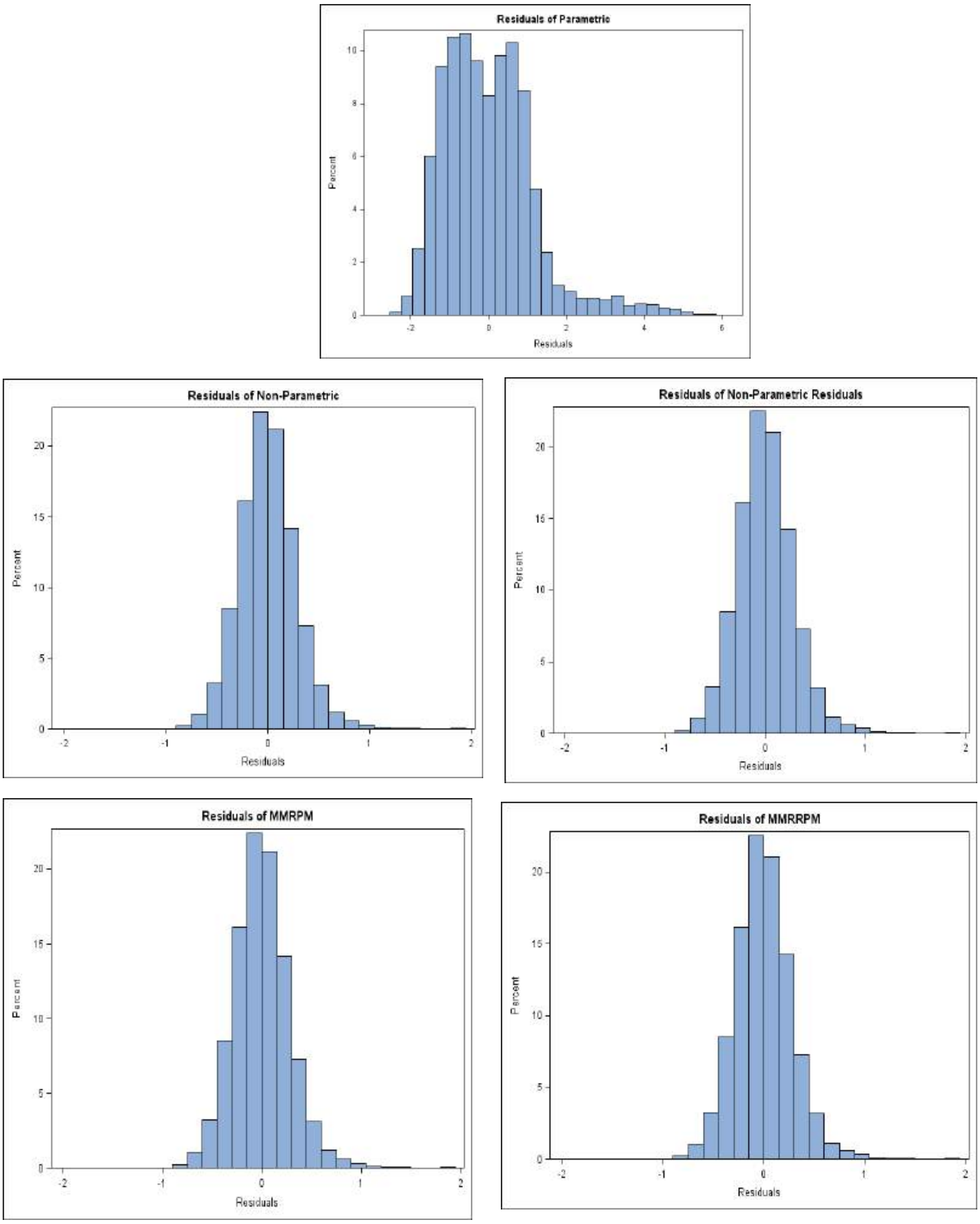

Figure 6.8. Vertical Density Profile Residuals: (a). Parametric fit, (b). Non-Parametric fit on raw data, (c). Non-Parametric fit on Residuals, (d). Semi-parametric fit MMRPM, (e). Semi-parametric fit MMRRPM

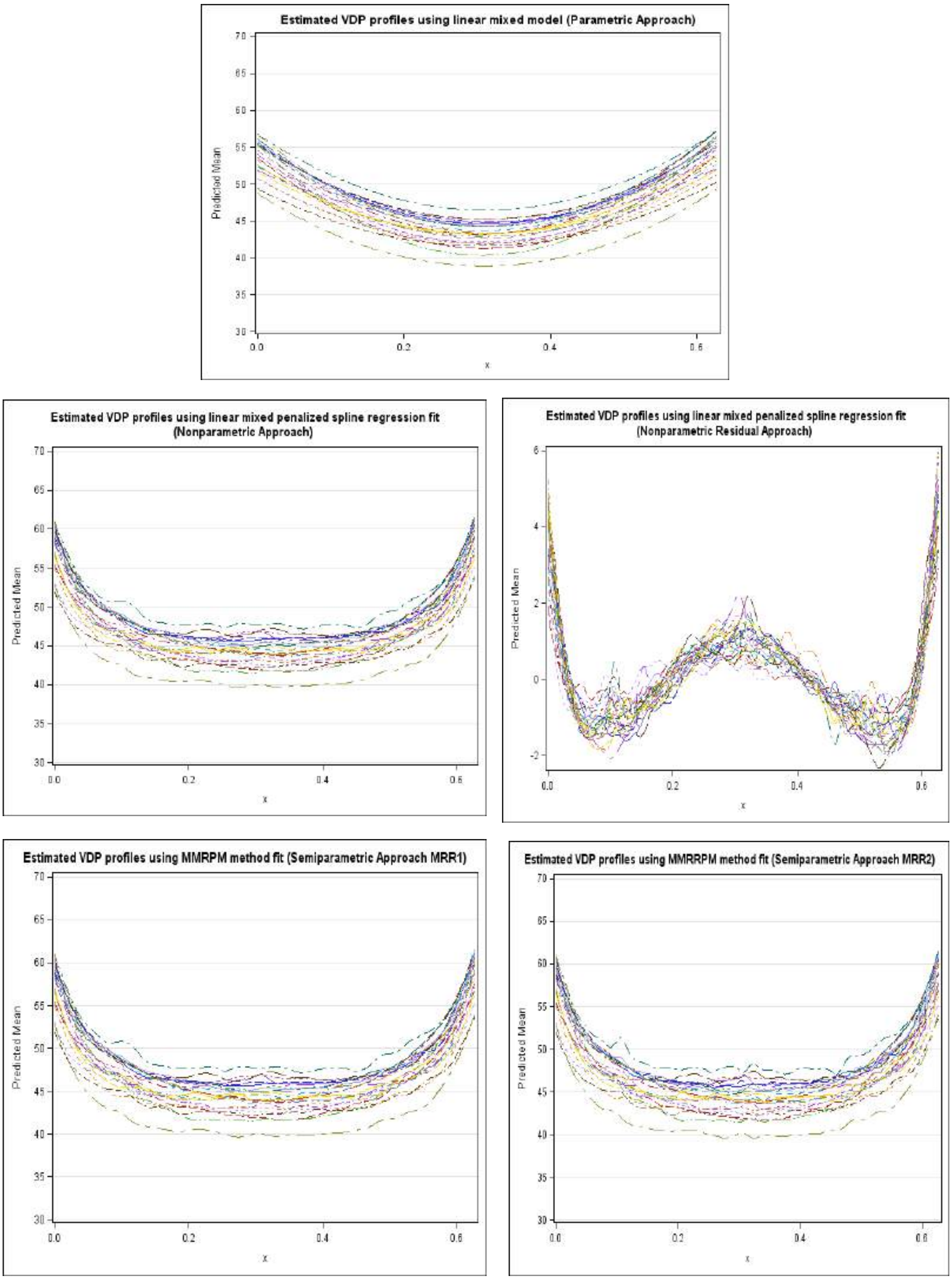

Figure 6.9. Vertical Density Profiles: (a). Parametric fit, (b). Non-Parametric fit on raw data, (c). Non-Parametric fit on Residuals, (d). Semi-parametric fit MMRPM, (e). Semi-parametric fit MMRRPM

The fitted profiles shown in *Figure (6.9),* as it can be seen, the parametric approach provides a smooth fit for the data while in fact, the data has fluctuations and therefore this approach is not capable to model this profile data. The non-parametric, semi-parametric techniques MMRPM and MMRRPM present the fluctuations and consider peaks and dips, thus they fit the model better.

Furthermore, the mean square error MSE for each technique was computed:

- Parametric MSE = 0.181
- Nonparametric MSE = 0.0435
- Nonparametric residuals MSE = 0.0305
- MMRPM MSE = 0.0435
- MMRRPM MSE = 0.0305

The parametric fit has the highest MSE compared to other techniques, the non-parametric and the semi-parametric techniques have less MSE than the parametric technique, the non-parametric residuals and MMRRPM techniques have the least MSE.

As what was done in the previous example, the upper control limits for the proposed MEWMA and MCUSUM charts for all the five techniques were found to achieve the desired ARL_0=200. The performances of these charts are examined by considering the first sixteen profiles as in-control and the last eight profiles were used to show an out-of-control condition for a slope shift coefficient of 0.5. The sensitivity of the proposed MEWMA and MCUSUM charts is represented in *Figure (6.10)* and *Figure (6.11),* respectively.

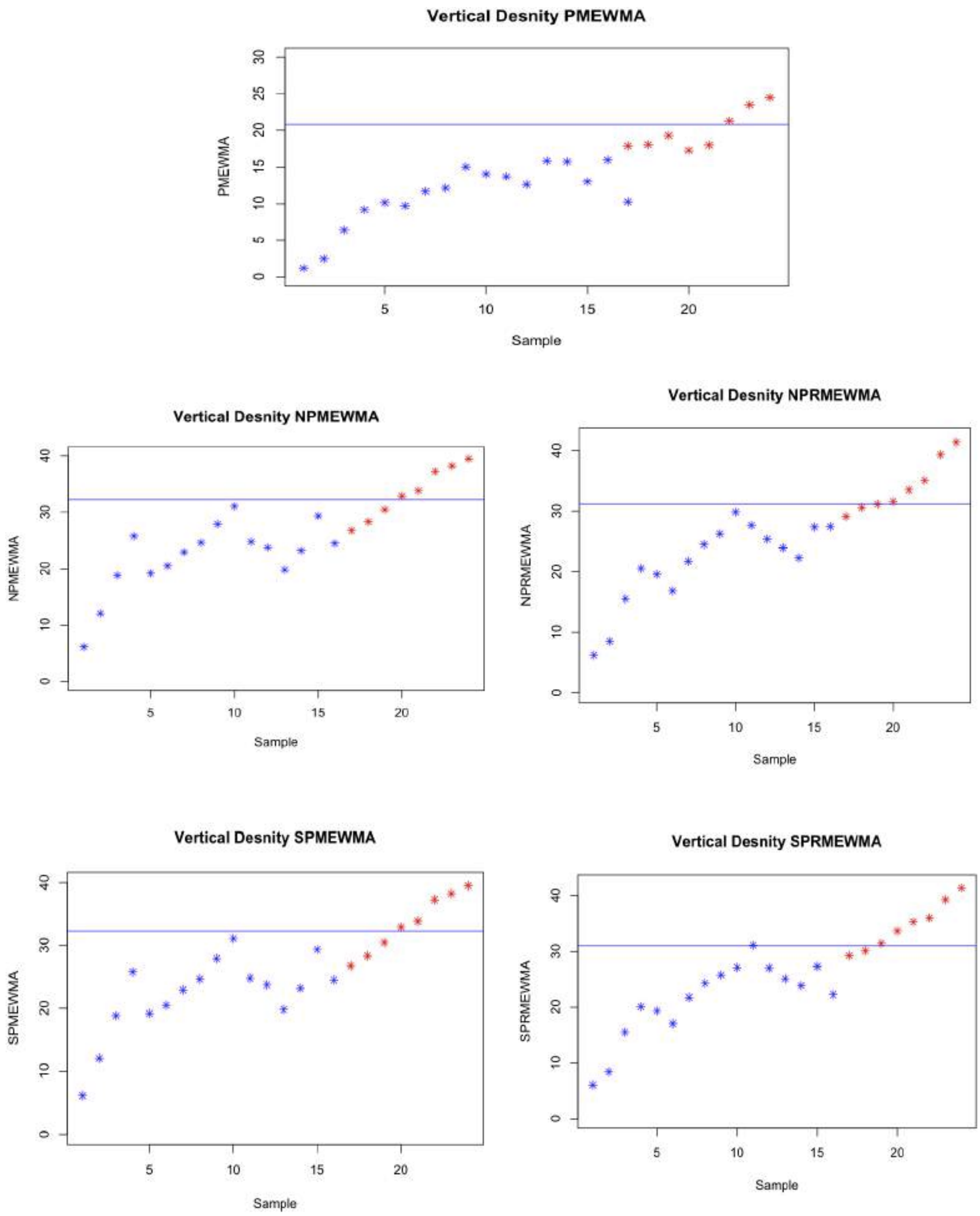

Figure 6.10. Vertical Density MEWMA: (a). Parametric MEWMA, (b). Non-Parametric MEWMA, (c). Non-Parametric Residuals MEWMA, (d). Semi-parametric MMRPM MEWMA, (e). Semi-parametric MMRRPM MEWMA

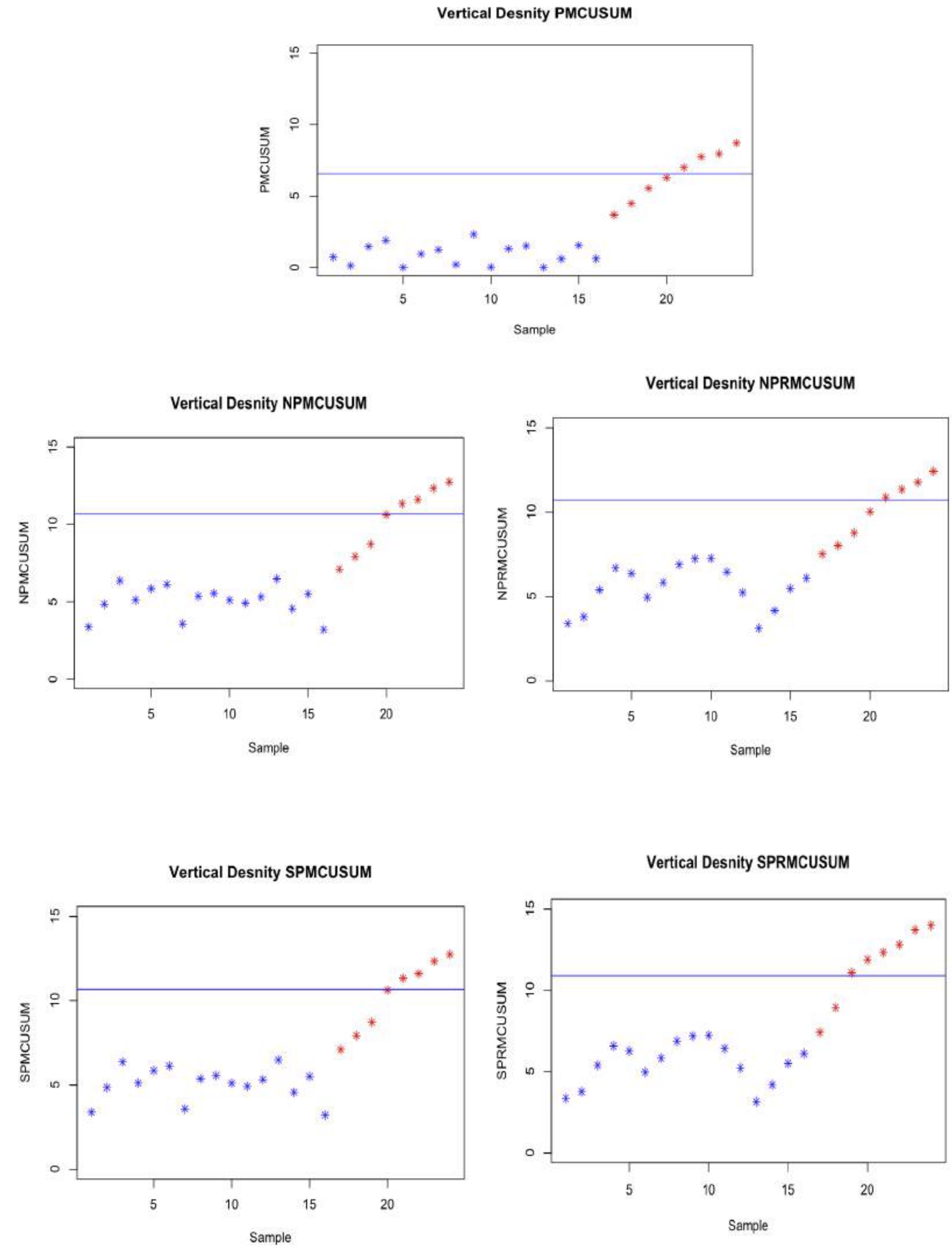

Figure 6.11: Vertical Density MCUSUM: (a). Parametric MCUSUM, (b). Non-Parametric MCUSUM, (c). Non-Parametric Residuals MCUSUM, (d). Semi-parametric MMRPM MCUSUM, (e). Semi-parametric MMRRPM MCUSUM

The sensitivity of MEWMA and MCUSUM charts are represented in *Figure (6.10)* and *Figure (6.11),* respectively. As mentioned previously, the in-control samples are represented as blue points, while the out-of-control samples are represented as red points. From both figures, it can be seen that the parametric charts (PMEWMA & PMCUSUM) needed five and four profiles respectively, to detect an out-of-control process. The non-parametric charts (NPMEWMA & NPRMEWMA) needed almost three profiles but using the non-parametric MCUSUM (NPMCUSUM & NPRMCUSUM) charts. We can also see that on the fourth out-of-control profile, the shift was detected; this may lead to better performance of the non-parametric charts compared to the parametric charts. Furthermore, the semi-parametric MMRPM charts (SPMEWMA & SPMCUSUM) had the same performances as the non-parametric (NPMEWMA & NPMCUSUM) charts and this is because the mixing parameter (explained in chapter 3) was estimated to be one. We can see that, the other semi-parametric MMRRPM (SPRMEWMA & SPRMCUSUM) charts needed two profiles only to detect an out-of-control process and there is one false alarm in SPRMEWMA. We can conclude that the sensitivity of the MMRRPM technique is more than other techniques in this example.

6.3: Automobile Engines Dataset

The Automobile Engine Dataset is described as a polynomial model and it was adapted from Amiri et al. (2010). The relationship between the response variable (Y): torque produced by the engine and the independent variable (x): speed in revolutions per minute (RPM), is the quality characteristic that was monitored. The dataset contained 26 engines and the torque for each engine was recorded at 14 different points of RPM. *Figure (6.12)* shows the raw engine data, as it can be seen, the curves are not smooth and thus, the parametric technique may not perfectly fit the data. As done previously, the non-parametric and semi-parametric techniques will be used to fit this data and to compare the performances of all charts.

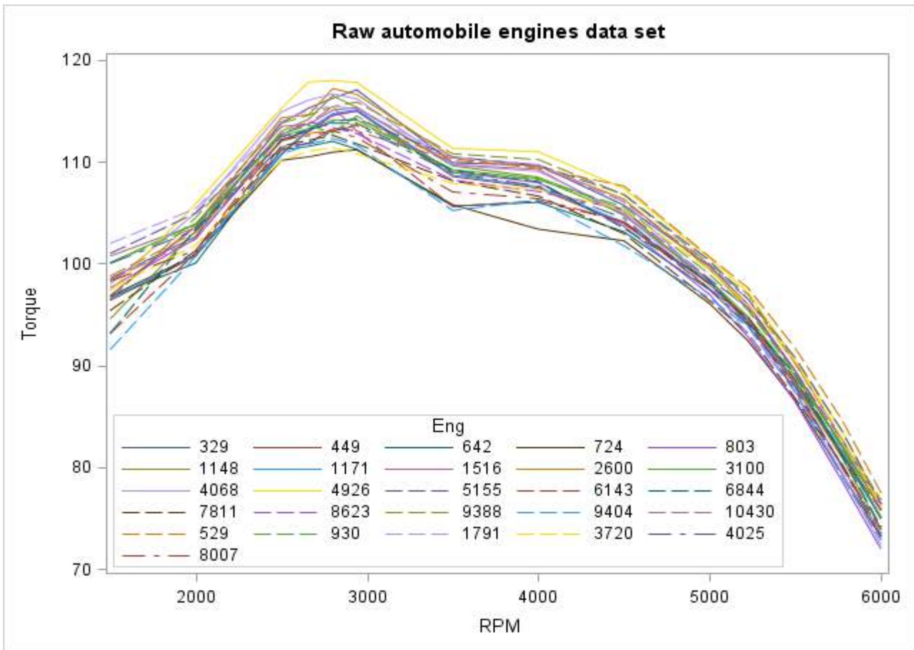

Figure 6.12. Raw dataset for Automobile engine

As the two previous examples, the parametric, non-parametric and semi-parametric techniques will be conducted to compare their performances. A quadratic model for the data was fitted as shown in *equation (6.3)* to estimate the in-control limits for each chart and we considered fitting a modified mean-corrected model such that: ${x_{ij}}^* = x_{ij} - \bar{x}$ represents the RPM mean-corrected vector. Furthermore, AR(1) structure was considered for accounting the correlation of errors between profiles, as Amiri, Jensen, and Kazemzadeh have assumed.

$$y_{ij} = 111.21 + b_{0j} + (-0.00599 + b_{1j}){x_{ij}}^* + (-0.00000494){x_{ij}}^{*2} + \varepsilon_{ij} \quad (6.3)$$

$$\varepsilon_{ij} = 0.4499\varepsilon_{i-1j} + a_{ij}$$

where, $b_{0j} \sim N(0,0.4652)$, $b_{1j} \sim N(0,0.000097)$ and $a_{ij} \sim N(0,0.0543)$

Also, the normality assumption of residuals was checked as shown in *Figure (6.13)*. For parametric estimation, we can see that the residuals are slightly left-skewed and this is an indication for un-proper fitting for the data and a better fit for the non-parametric and semi-parametric techniques as the residuals are almost normal.

Using the parametric, the two non-parametric and the two semi-parametric techniques, profiles were fitted as shown in *Figure (6.14)*.

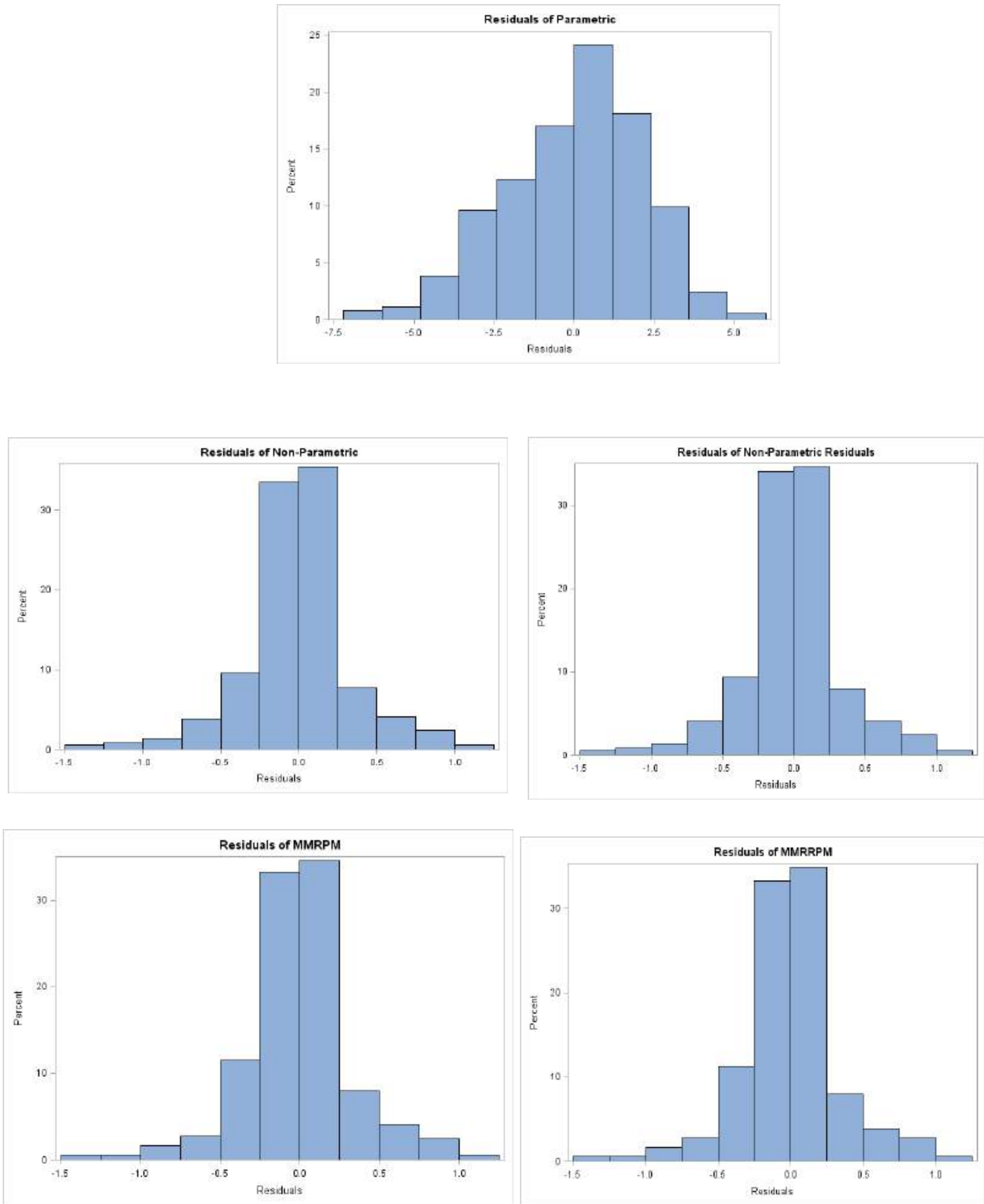

Figure 6.13. Automobile Engine Profile Residuals: (a). Parametric fit, (b). Non-Parametric fit on raw data, (c). Non-Parametric fit on Residuals, (d). Semi-parametric fit MMRPM, (e). Semi-parametric fit MMRRPM

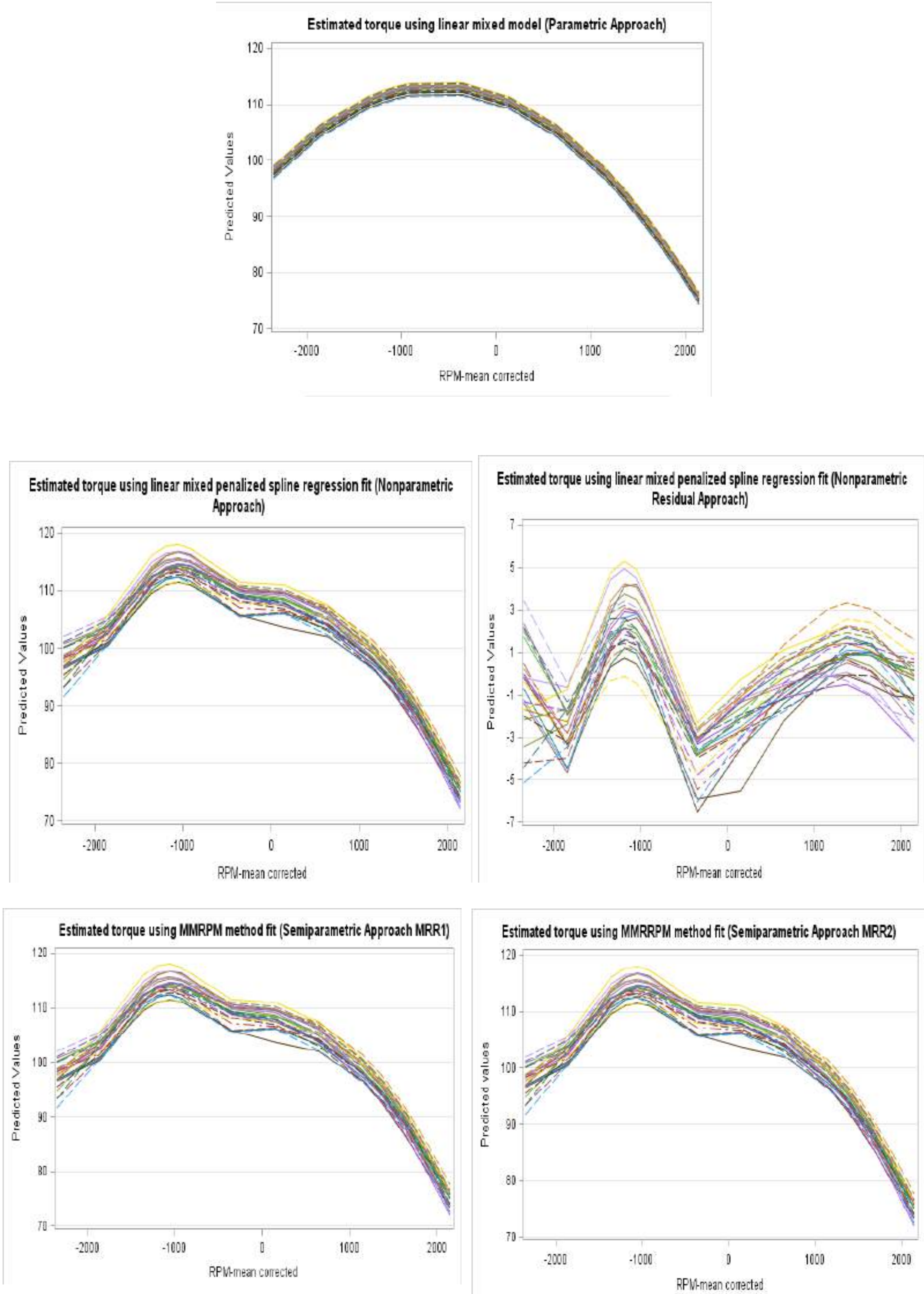

Figure 6.14. Automobile Engine Profiles: (a). Parametric fit, (b). Non-Parametric fit on raw data, (c). Non-Parametric fit on Residuals, (d). Semi-parametric fit MMRPM, (e). Semi-parametric fit MMRRPM

From *Figure (6.14),* it can be seen that the parametric fit does not capture the fluctuations and other profile features, while we can see that the non-parametric (NP) approach and the semi-parametric (MMRPM & MMRRPM) approaches are monotonic and capture almost all the bends and features of the data and thus these techniques have a better fit of the data compared to the parametric technique.

The mean square error (MSE) was computed for all the five techniques and the results are as follows:

- Parametric MSE = 4.8210
- Nonparametric MSE = 0.1169
- Nonparametric residuals MSE = 0.1171
- MMRPM MSE = 0.1162
- MMRRPM MSE = 0.1164

We notice that the MSE of the parametric approach is the highest compared to others, the semi-parametric MMRPM approach had the lowest MSE, but in fact it is close to the other semi-parametric MMRRPM approach. Moreover, the non-parametric NP technique had a lower MSE compared to the other non-parametric residuals technique. We can conclude that the semi-parametric techniques are superior to the P, NP and NPR approaches according to their low MSE values.

Finally, the upper control limits for each chart were constructed using *equation (6.3),* twenty profiles of the data were used as in-control, while a shift of 0.5 was inserted to the last six profiles to result in an out-of-control profiles. The MEWMA and MCUSUM charts are shown in *Figures (6.15) & (6.16)* below:

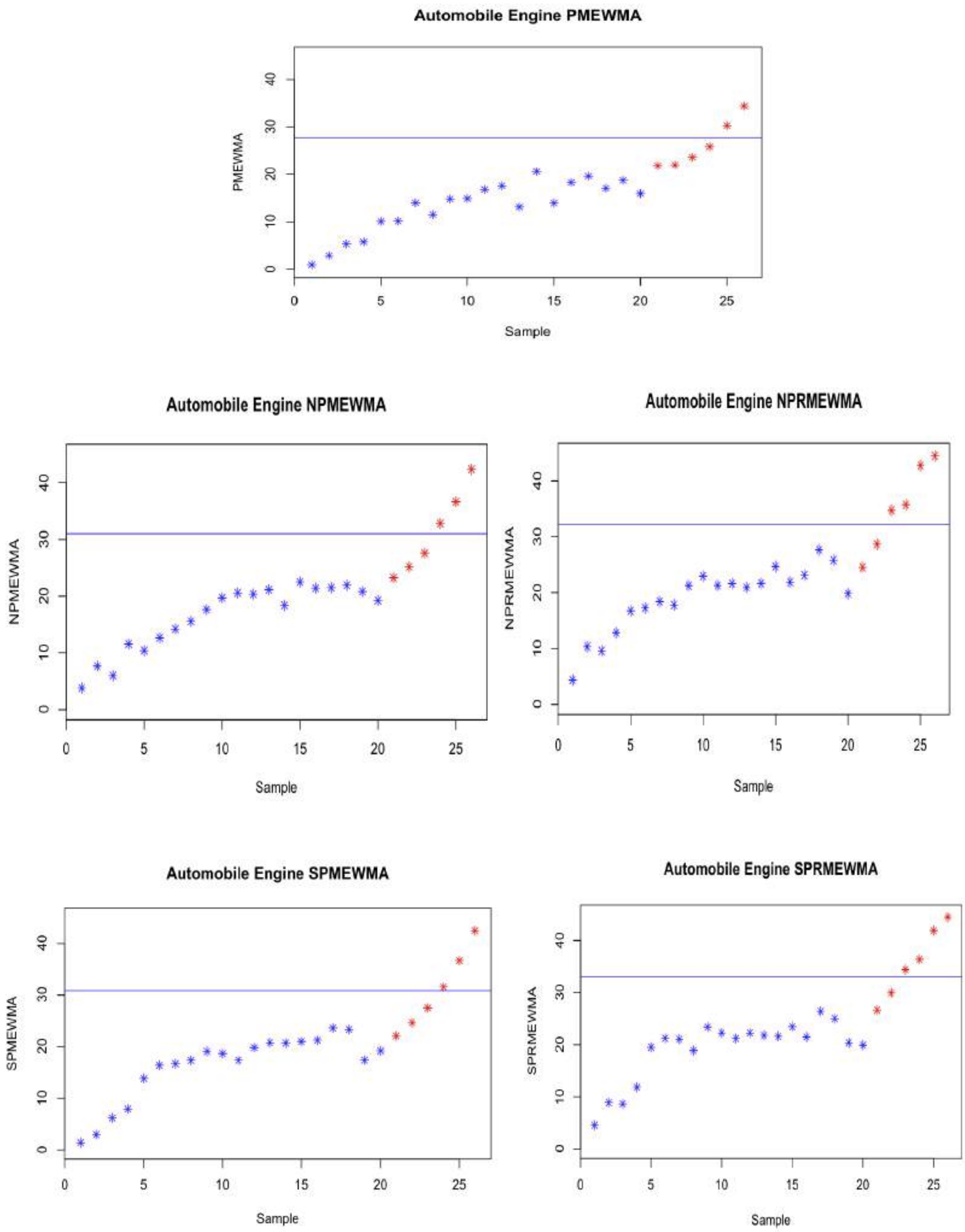

Figure 6.15. Automobile Engine MEWMA: (a). Parametric MEWMA, (b). Non-Parametric MEWMA, (c). Non-Parametric Residuals MEWMA, (d). Semi-parametric MMRPM MEWMA, (e). Semi-parametric MMRRPM MEWMA

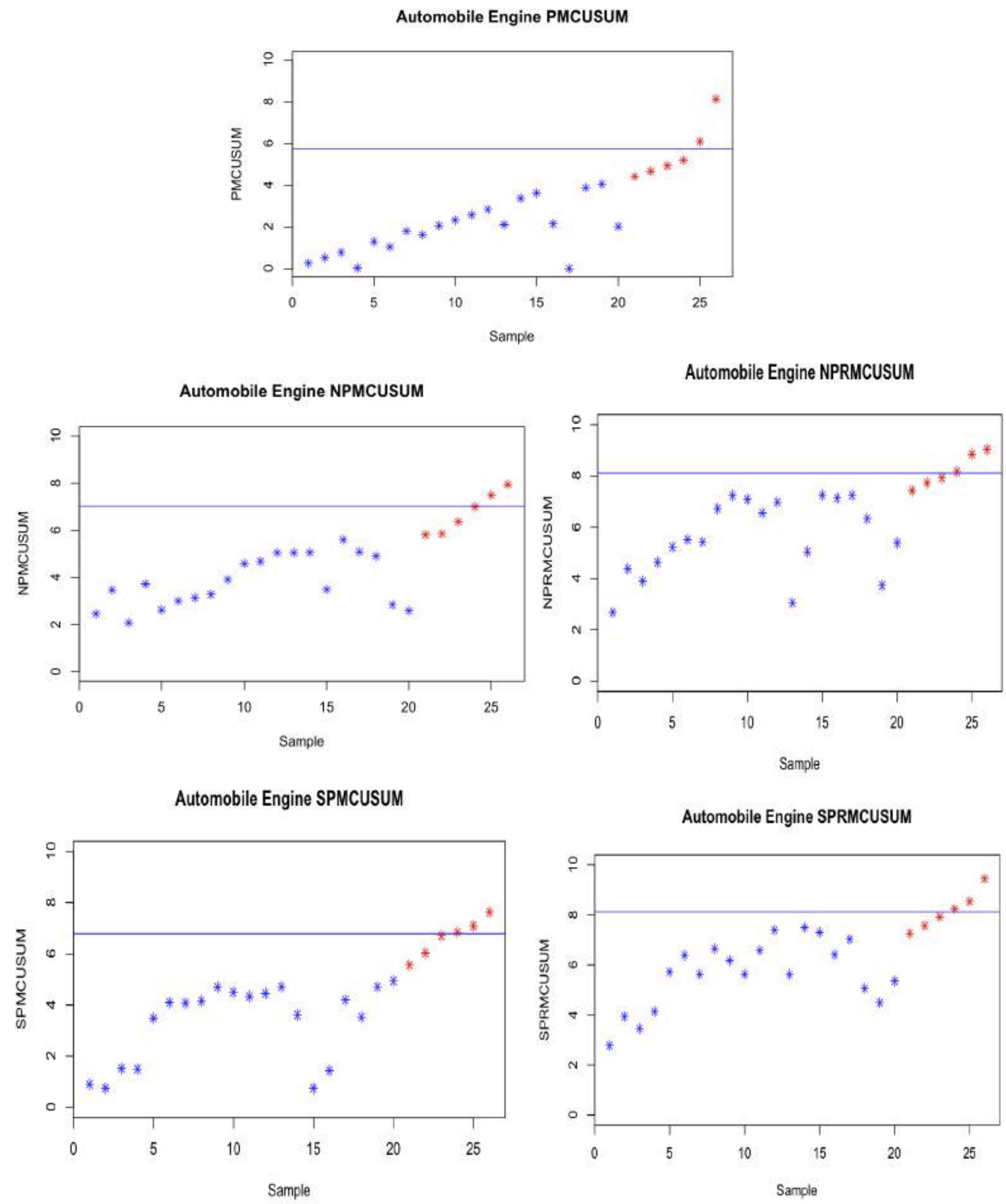

Figure 6.16. Automobile Engine MCUSUM: (a). Parametric MCUSUM, (b). Non-Parametric MCUSUM, (c). Non-Parametric Residuals MCUSUM, (d). Semi-parametric MMRPM MCUSUM, (e). Semi-parametric MMRRPM MCUSUM

Both *Figures (6.15)* & *(6.16)* represent MEWMA and MCUSUM charts for automobile engine data, respectively. The parametric MEWMA and MCUSUM charts had the worst performances in detecting the shift as they consumed four profiles, while the non-parametric techniques had better performances. The NPR charts performed better than the NP charts as they required fewer profiles for detecting the shift. Furthermore, we can see that the performance of MMRPM is close to the NP performance, yet SPMCUSUM performed better than the NPMCUSUM chart. Moreover, MMRRPM charts had superior performances compared to all other techniques, especially for SPRMEWMA, as only two profiles were used to detect an out-of-control process. We can conclude that for this dataset, it is recommended to use the MMRRPM technique and then the NPR technique.

CHAPTER 7: SUMMARY & CONCLUSION

The statistical process control (SPC) was introduced by Walter A. Shewhart in 1920s, where it is a technique used to monitor the changes in a process by using different statistical methods and one of these methods is the control chart, where it is used to improve the quality of the process. Any process passes by two phases, Phase I, which analyzes historical data to construct trial control limits and to eliminate out-of-control points. While Phase II is an "online" monitoring process. The quality of a process can be represented by a relationship between the response variable and explanatory variables; this relationship is known as "profile", and monitoring such a relation is called "profile monitoring." Profile monitoring became so popular nowadays and it can be applied to many different fields such as medical, industrial, educational and productional.

Many studies were conducted for monitoring profiles in Phase I and Phase II. Some studies monitored linear profiles while others monitored non-linear profiles, moreover, some studies considered different scenarios such as correlated, uncorrelated, balanced and unbalanced data. Two studies were conducted for monitoring linear mixed model profiles; the first study was conducted by Abdelsalam (2009), where they proposed MMRPM for monitoring profiles in Phase I, MMRPM is a model robust regression estimation technique and it is based on MRR1 that was proposed by Einsporn and Birch (1993). The MRR1 is a semi-parametric technique that combines the data fitted using the parametric technique with the data fitted using the non-parametric technique. The other study was proposed by Siddiqui and Abdelsalam (2019), where they proposed MMRRPM for monitoring profiles in Phase I, it is also a model robust regression estimation technique based on MRR2 that was proposed by Mays et al. (2000). The MRR2 combines the data fitted using a parametric technique with the non-parametric fitted residuals that were obtained from the parametric technique. In both studies, they compared the abilities of their proposed methods in detecting an out-of-control signal and compared their efficiencies.

This study considers fitting linear mixed models (LMM) according to their flexibility and their consideration of both fixed and random effects, or in other words, LMM considers similarities and differences between profiles. These linear mixed profiles are fitted using five techniques, which are:

- The parametric technique (P using Generalized Least Square Method)
- The non-Parametric technique for raw data (NP using Penalized Spline Method)
- The non-Parametric technique for residuals obtained from the parametric estimation (NPR using Penalized Spline Method)
- The semi-Parametric technique (MMRPM using MRR1 Method)
- The semi-Parametric technique (MMRRPM using MRR2 Method)

Parametric techniques usually fit the model, assuming that its distribution is known, but if the model is misspecified, this technique will be misleading, causing biased parameter estimation. Non-Parametric techniques are alternative approaches are used when the distribution is unknown; these techniques are more variable compared to the parametric technique, but they often result in estimates with large variances. Therefore, semi-parametric techniques are used to solve the problems of the previous approaches by combining parametric and non-parametric fits by taking only the advantages of each technique.

The performances of these techniques mentioned above were measured by monitoring linear mixed effect profiles in Phase II by using MEWMA and MCUSUM control charts. We have studied the sensitivity of these charts in detecting slope shifts of mixed effect profiles. Several Monte-Carlo simulation studies were conducted to compare the proposed techniques to the parametric using the simulated integrated mean square error (SIMSE). Also, the performances of the parametric (P), the two non-parametric (NP & NPR) and the two semi-parametric (MMRPM & MMRRPM) techniques under different model misspecification levels, different correlations, different profile sizes and different sample sizes were evaluated. From SIMSE results, we have found that as the sample size n increases per profile, the SIMSE decreases. Furthermore, when there is no model misspecification, the two semi-parametric (MMRPM & MMRRPM) techniques had close results to the parametric technique. While they get close to the non-parametric (NP & NPR) techniques as the model misspecification increases. Also, for a small sample size, both the non-parametric (NP & NPR) had the same performances, but for large samples, the NPR technique outperformed NP technique. The two semi-parametric (MMRPM & MMRRPM) techniques performed better than P and NPR techniques, where in most cases, the MMRRPM techniques had lower SIMSE compared to the MMRPM technique and thus it outperformed it.

Moreover, the sensitivity of all different techniques was checked using Average Run Length (ARL) and extra quadratic loss (EQL) across all different scenarios, it was found that both MMRPM and MMRRPM techniques showed superior performances in detecting different amounts of slope shifts compared to the parametric technique and the non-parametric techniques. Furthermore, as the model misspecification level increases, the non-parametric techniques showed a better performance compared to the parametric technique and thus, the parametric technique may not appropriately fit the data well, but yet the semi-parametric charts were better than the non-parametric charts and thus we recommend the use of the semi-parametric techniques as they are robust to model misspecification and have higher abilities in detecting different amounts of shifts.

In addition to Monte Carlo simulation, three real-applications were presented in this study to check the performances of the proposed charts; one was the apple diameter dataset, the second was the vertical density dataset and the third was the automobile engine dataset. For apple diameter dataset, it was found that the two semi-parametrics MEWMA & MCUSUM charts are more sensitive in detecting shifts compared to other charts. Also, the sensitivity of parametric and non-parametric charts were close to each other. For the vertical density dataset, the semi-parametric (MMRPM) technique had the same performance as the non-parametric (NP) technique, while the other semi-parametric (MMRRPM) technique had the best performance in detecting an out-of-control process. Moreover, using the third dataset, which is the automobile engine dataset, we recommend using the semi-parametric (MMRRPM) technique, in addition to the non-parametric NPR charts.

In conclusion, we recommend using the semi-parametric techniques as they had the lowest SIMSE values in simulation and the lowest MSE values in real data applications, in addition to their better and faster out-of-control process detection.

FUTURE RECOMMENDATIONS

This study focuses on monitoring the slope of the profiles in Phase II. Thus future studies may consider monitoring the intercept or the variance-covariance matrix. Furthermore, the MEWMA and MCUSUM charts were based on the estimated random effects; for future studies, these charts can be based on the fitted values instead of the estimated random-effects. Also, it can be applied to different real-life fields other than the ones considered in this study, such as educational or medical fields.

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APPENDIX

Appendix Table 1.1. MEWMA ATS for un-correlated (ρ =0.0) Profile Datasets (m=300 , n=10 , l=40)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	340.28	357.94	354.32	322.78	294.82
	0.10	300.96	313.78	317.20	278.86	250.80
	0.15	273.24	282.06	281.30	238.70	216.58
	0.20	226.84	245.76	240.76	210.32	181.8
	0.25	191.56	213.00	216.70	175.42	156.90
	0.30	145.88	181.32	179.68	139.08	128.58
	0.50	74.40	97.40	96.50	71.10	62.50
0.25	0.00	398.76	349.50	350.60	303.54	298.14
	0.05	343.54	313.74	310.68	271.00	261.66
	0.10	294.98	286.06	285.28	241.62	239.24
	0.15	267.26	244.88	243.56	212.86	206.64
	0.20	225.34	200.18	207.28	181.76	179.9
	0.25	193.66	164.04	167.70	132.22	129.8
	0.30	141.96	115.52	117.96	94.48	90.56
	0.50	77.42	60.20	63.06	53.44	49.44
0.50	0.00	392.98	299.74	295.78	270.84	271.26
	0.05	338.10	264.06	262.40	240.46	238.10
	0.10	282.18	233.88	231.46	215.82	209.18
	0.15	255.94	206.44	204.96	181.00	172.64
	0.20	224.32	179.16	179.04	144.38	136.92
	0.25	190.70	144.66	141.32	100.74	99.28
	0.30	156.70	101.26	105.5	76.12	70.62
	0.50	80.74	48.00	47.96	40.26	38.98
0.75	0.00	393.26	273.48	269.98	251.72	246.94
	0.05	348.20	238.64	235.58	214.58	207.06
	0.10	317.46	205.58	201.58	185.38	180.06
	0.15	281.08	170.80	174.80	148.62	143.32
	0.20	246.56	138.50	140.62	107.26	104.62
	0.25	200.44	109.14	111.5	79.98	76.44
	0.30	156.18	76.16	74.68	53.54	49.46
	0.50	85.38	40.62	39.12	26.52	22.02
1.00	0.00	397.38	257.92	252.20	221.68	208.84
	0.05	338.92	209.10	211.20	192.32	181.08
	0.10	285.22	180.98	180.30	167.68	156.66
	0.20	220.40	124.54	120.12	84.86	74.00
	0.25	187.16	89.16	86.06	54.56	49.98
	0.30	141.2	59.78	56.22	30.08	23.58
	0.50	71.18	32.42	30.36	14.94	13.42

Appendix Table 1.2. MEWMA ATS for un-correlated (ρ =0.0) Profile Datasets (m=300 , n=20 , l=40)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	347.60	352.88	352.78	325.86	317.86
	0.10	301.70	308.58	307.06	290.72	279.20
	0.15	269.68	280.56	283.96	255.48	245.52
	0.20	234.96	249.58	253.96	224.20	213.94
	0.25	196.70	212.06	211.08	184.60	175.68
	0.30	150.28	168.18	167.06	140.44	132.58
	0.50	87.60	103.04	98.20	77.24	69.70
0.25	0.00	399.78	358.54	356.74	310.34	306.98
	0.05	336.34	324.40	320.98	269.92	259.12
	0.10	304.12	284.84	281.5	230.64	228.42
	0.15	277.04	241.88	242.78	200.06	201.26
	0.20	225.26	207.92	210.72	175.88	168.42
	0.25	186.70	169.16	167.68	130.94	122.76
	0.30	153.78	131.42	129.08	93.42	85.32
	0.50	80.26	68.92	65.18	51.48	43.56
0.50	0.00	398.80	331.74	327.08	293.94	285.06
	0.05	336.54	272.06	271.80	249.92	241.34
	0.10	309.40	238.30	235.96	215.76	213.06
	0.15	274.14	201.66	202.86	179.66	170.46
	0.20	231.38	169.98	175.84	149.00	138.42
	0.25	197.94	140.86	146.36	116.10	111.80
	0.30	158.32	116.26	120.88	79.04	77.34
	0.50	89.54	60.92	62.18	43.76	33.36
0.75	0.00	398.92	294.22	293.68	260.32	266.98
	0.05	345.48	244.84	249.14	233.94	238.94
	0.10	300.26	207.18	210.98	195.80	200.88
	0.15	264.68	174.86	176.90	161.68	168.40
	0.20	221.04	147.02	150.50	132.46	141.56
	0.25	197.82	118.20	112.68	107.8	111.24
	0.30	154.12	92.68	94.08	69.00	72.90
	0.50	92.14	55.02	50.68	41.82	43.44
1.00	0.00	393.58	276.34	270.02	224.60	220.18
	0.05	341.02	232.96	229.74	200.42	191.12
	0.10	298.94	190.60	186.96	164.24	159.06
	0.15	265.42	157.52	161.88	131.88	127.56
	0.20	236.34	126.22	135.02	94.70	90.34
	0.25	202.36	99.90	92.30	69.76	61.34
	0.30	161.18	74.40	71.96	41.18	39.18
	0.50	94.74	44.30	41.68	27.84	23.26

Appendix Table 1.3. MEWMA ATS for un-correlated (ρ =0.0) Profile Datasets (m=600 , n=10 , l=70)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	347.60	352.88	352.78	325.86	317.86
	0.10	301.70	308.58	307.06	290.72	279.20
	0.15	269.68	280.56	283.96	255.48	245.52
	0.20	234.96	249.58	253.96	224.20	213.94
	0.25	196.70	212.06	211.08	184.60	175.68
	0.30	150.28	168.18	167.06	140.44	132.58
	0.50	87.60	103.04	98.20	77.24	69.70
0.25	0.00	399.78	358.54	356.74	310.34	306.98
	0.05	336.34	324.40	320.98	269.92	259.12
	0.10	304.12	284.84	281.5	230.64	228.42
	0.15	277.04	241.88	242.78	200.06	201.26
	0.20	225.26	207.92	210.72	175.88	168.42
	0.25	186.70	169.16	167.68	130.94	122.76
	0.30	153.78	131.42	129.08	93.42	85.32
	0.50	80.26	68.92	65.18	51.48	43.56
0.50	0.00	398.80	331.74	327.08	293.94	285.06
	0.05	336.54	272.06	271.80	249.92	241.34
	0.10	309.40	238.30	235.96	215.76	213.06
	0.15	274.14	201.66	202.86	179.66	170.46
	0.20	231.38	169.98	175.84	149.00	138.42
	0.25	197.94	140.86	146.36	116.10	111.80
	0.30	158.32	116.26	120.88	79.04	77.34
	0.50	89.54	60.92	62.18	43.76	33.36
0.75	0.00	398.92	294.22	293.68	260.32	266.98
	0.05	345.48	244.84	249.14	233.94	238.94
	0.10	300.26	207.18	210.98	195.80	200.88
	0.15	264.68	174.86	176.90	161.68	168.40
	0.20	221.04	147.02	150.50	132.46	141.56
	0.25	197.82	118.20	112.68	107.8	111.24
	0.30	154.12	92.68	94.08	69.00	72.90
	0.50	92.14	55.02	50.68	41.82	43.44
1.00	0.00	393.58	276.34	270.02	224.60	220.18
	0.05	341.02	232.96	229.74	200.42	191.12
	0.10	298.94	190.60	186.96	164.24	159.06
	0.15	265.42	157.52	161.88	131.88	127.56
	0.20	236.34	126.22	135.02	94.70	90.34
	0.25	202.36	99.90	92.30	69.76	61.34
	0.30	161.18	74.40	71.96	41.18	39.18
	0.50	94.74	44.30	41.68	27.84	23.26
UCL		31.58	45.13	45.10	31.58	31.58

Appendix Table 1.4. MEWMA ATS for un-correlated (ρ =0.0) Profile Datasets (m=600 , n=20 , l=70)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	340.96	347.88	352.32	298.58	310.38
	0.10	296.98	301.00	302.76	249.56	263.86
	0.15	260.70	268.94	272.48	205.58	222.70
	0.20	228.86	232.62	234.46	175.94	194.86
	0.25	197.30	205.64	202.48	146.00	163.64
	0.30	165.86	169.66	168.48	104.14	112.66
	0.50	78.60	89.44	87.68	43.10	60.68
0.25	0.00	396.04	318.70	328.18	283.74	303.14
	0.05	328.12	285.74	298.98	255.94	273.58
	0.10	295.68	250.58	254.44	229.54	249.52
	0.15	246.62	220.54	229.80	201.12	213.42
	0.20	216.60	196.30	198.96	174.72	180.94
	0.25	187.60	162.66	167.32	147.88	155.46
	0.30	158.70	131.46	132.32	114.44	120.42
	0.50	78.28	77.02	77.08	49.02	50.34
0.50	0.00	388.44	279.72	281.66	240.58	251.26
	0.05	340.36	251.18	252.88	214.28	211.46
	0.10	304.82	221.52	223.18	191.96	185.34
	0.15	251.78	196.20	197.68	161.66	161.48
	0.20	203.64	168.40	171.02	128.84	121.88
	0.25	172.42	140.54	142.96	78.58	69.58
	0.30	144.30	116.30	115.64	46.08	41.24
	0.50	77.38	69.44	68.26	27.38	23.56
0.75	0.00	398.54	244.58	244.02	216.64	209.44
	0.05	328.72	220.40	221.52	176.92	173.56
	0.10	294.10	201.68	201.22	148.46	147.08
	0.15	257.18	177.92	178.40	116.40	108.96
	0.20	219.96	152.46	153.50	96.26	84.80
	0.25	190.26	127.50	125.04	65.88	60.60
	0.30	143.12	101.88	99.64	41.36	39.94
	0.50	73.48	55.78	56.62	20.58	20.70
1.00	0.00	399.52	215.88	217.42	187.04	185.88
	0.05	344.42	191.54	190.70	154.98	153.76
	0.10	318.90	160.34	163.38	126.50	123.60
	0.15	255.76	135.56	135.34	100.54	93.46
	0.20	216.88	110.4	108.22	74.44	69.54
	0.25	186.78	86.18	74.30	52.04	53.18
	0.30	143.92	62.66	53.14	31.90	27.12
	0.50	77.66	33.98	28.58	15.34	15.72
UCL		31.10	45.32	45.45	31.10	31.10

Appendix Table 1.5. MCUSUM ATS for un-correlated (ρ =0.0) Profile Datasets (m=300 , n=10 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	336.98	357.90	352.46	314.66	302.74
	0.10	294.58	305.08	301.56	268.54	252.62
	0.15	261.04	271.62	267.74	221.06	214.94
	0.20	218.88	228.10	233.96	194.04	189.92
	0.25	180.64	201.88	205.76	161.36	158.92
	0.30	150.46	161.56	167.80	130.86	126.38
	0.50	77.80	84.12	84.82	67.88	59.54
0.25	0.00	395.18	357.28	354.06	321.8	320.14
	0.05	340.32	306.62	301.02	263.48	273.46
	0.10	296.58	260.40	258.18	236.44	229.18
	0.15	261.08	237.96	231.06	209.24	199.50
	0.20	237.06	206.82	205.94	180.52	174.42
	0.25	193.18	178.58	182.16	156.04	147.28
	0.30	148.72	136.82	133.00	119.12	120.18
	0.50	82.56	70.98	71.62	56.64	55.08
0.50	0.00	398.98	326.48	319.88	301.78	296.12
	0.05	338.40	277.20	279.52	258.14	251.46
	0.10	300.10	244.56	240.96	221.60	213.18
	0.15	270.22	199.44	201.48	186.80	178.76
	0.20	227.70	161.90	171.38	148.50	143.8
	0.25	195.20	135.64	142.72	123.92	116.22
	0.30	156.14	108.26	104.08	95.12	90.94
	0.50	87.22	62.98	60.36	50.24	44.76
0.75	0.00	393.44	310.04	300.44	280.20	275.04
	0.05	326.80	244.90	246.72	228.00	227.08
	0.10	297.48	215.34	218.92	197.28	190.78
	0.15	273.02	190.86	188.92	153.6	140.22
	0.20	240.20	139.36	140.34	121.44	111.90
	0.25	198.86	101.00	96.86	87.32	80.70
	0.30	141.22	77.60	71.36	61.22	53.34
	0.50	80.20	42.50	41.02	31.24	26.84
1.00	0.00	398.44	276.20	273.20	249.84	239.86
	0.05	313.12	232.92	231.74	226.04	217.94
	0.10	274.02	194.74	201.58	185.84	175.64
	0.15	251.42	157.42	168.36	147.62	139.64
	0.20	220.36	113.88	133.76	101.42	101.36
	0.25	194.24	88.92	94.46	69.3	63.34
	0.30	151.06	65.30	59.68	32.78	31.12
	0.50	74.18	34.14	32.30	10.46	9.52
UCL		7.94	15.01	15.11	7.94	7.94

Appendix Table 1.6. MCUSUM ATS for un-correlated (ρ =0.0) Profile Datasets (m=300 , n=20 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	337.34	346.54	338.64	308.52	306.40
	0.10	290.28	300.22	288.80	280.76	276.80
	0.15	241.90	261.20	253.5	231.00	231.90
	0.20	209.36	225.34	219.64	198.84	189.18
	0.25	169.14	198.64	192.70	160.98	152.80
	0.30	143.16	156.44	154.80	129.00	124.58
	0.50	81.00	93.84	95.28	76.80	69.04
0.25	0.00	399.06	361.36	358.42	331.2.00	322.92
	0.05	340.22	314.76	309.48	281.38	271.46
	0.10	310.76	292.30	281.24	246.56	234.80
	0.15	282.88	245.20	243.68	210.98	199.18
	0.20	253.20	209.80	210.48	187.18	168.58
	0.25	211.38	181.44	179.02	160.52	141.04
	0.30	170.76	136.84	134.84	110.40	108.40
	0.50	84.74	69.60	65.40	58.80	44.46
0.50	0.00	396.80	310.600	307.86	281.06	275.04
	0.05	342.46	283.38	278.48	250.58	246.22
	0.10	309.20	257.18	255.66	220.44	212.48
	0.15	277.56	224.78	218.88	191.20	180.76
	0.20	250.22	187.56	191.44	158.28	150.66
	0.25	213.38	154.90	149.66	127.32	120.78
	0.30	170.74	120.74	117.16	95.14	85.14
	0.50	88.60	60.48	54.84	43.00	40.70
0.75	0.00	399.16	285.68	284.48	255.00	261.90
	0.05	340.24	244.74	247.48	220.16	225.26
	0.10	307.20	200.78	206.32	189.00	187.04
	0.15	277.18	170.70	172.64	148.56	153.46
	0.20	238.48	145.20	147.16	122.50	123.20
	0.25	203.54	110.82	107.78	86.98	89.38
	0.30	169.38	80.58	79.66	58.26	61.04
	0.50	85.20	42.80	45.66	34.38	33.68
1.00	0.00	397.62	275.68	273.66	227.98	225.20
	0.05	337.88	245.00	247.62	210.60	209.34
	0.10	298.10	202.98	209.58	183.06	178.60
	0.15	263.14	169.42	165.38	156.80	152.86
	0.20	220.44	131.38	128.54	124.54	120.98
	0.25	198.52	100.74	101.72	90.00	92.44
	0.30	160.22	68.66	66.54	57.00	51.50
	0.50	90.60	39.32	38.40	30.94	26.88
UCL		7.90	15.03	15.40	7.90	7.90

Appendix Table 1.7. MCUSUM ATS for uncorrelated (ρ =0.0) Profile Datasets (m=600 , n=10 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	345.42	348.40	352.62	288.72	301.90
	0.10	295.30	316.78	320.06	241.50	258.84
	0.15	236.48	272.56	285.78	203.82	217.10
	0.20	212.80	234.24	240.22	174.18	183.00
	0.25	183.30	196.88	202.44	140.94	153.46
	0.30	151.54	161.36	163.8	104.00	112.3
	0.50	72.90	75.96	84.76	57.46	58.16
0.25	0.00	398.68	296.30	300.3	274.22	282.58
	0.05	335.42	269.18	258.12	240.78	250.82
	0.10	307.78	231.70	224.36	201.06	210.94
	0.15	281.94	195.40	199.44	174.56	186.84
	0.20	253.82	160.42	171.88	138.2	154.2
	0.25	207.66	124.22	138.04	114.06	129.84
	0.30	156.22	99.42	105.88	82.96	96.26
	0.50	74.82	58.30	64.34	42.84	53.66
0.50	0.00	398.18	269.20	270.28	248.88	252.72
	0.05	373.64	236.80	237.28	206.56	212.28
	0.10	319.64	209.84	209.12	181.44	188.86
	0.15	282.04	181.00	182.88	155.12	160.6
	0.20	275.12	154.00	154.06	129.1	130.66
	0.25	220.36	123.16	125.44	94.14	105.9
	0.30	163.04	84.5	85.02	67.68	71.56
	0.50	84.52	50.72	50.90	33.26	35.36
0.75	0.00	391.06	256.62	278.94	248.24	251.18
	0.05	351.74	229.74	230.82	214.12	220.96
	0.10	323.52	198.98	199.06	190.50	195.24
	0.15	287.00	170.90	172.16	159.86	163.02
	0.20	248.18	140.52	141.70	115.44	120.68
	0.25	212.08	109.82	109.94	87.22	90.44
	0.30	168.12	67.34	67.70	54.76	60.16
	0.50	79.54	36.48	36.52	28.92	30.52
1.00	0.00	393.70	223.80	224.30	208.56	209.86
	0.05	354.16	199.84	200.66	178.50	184.08
	0.10	309.16	179.64	181.38	152.62	159.94
	0.15	271.32	155.14	157.16	128.52	129.60
	0.20	235.46	127.32	127.54	94.48	97.60
	0.25	193.68	97.08	98.26	64.96	69.52
	0.30	164.54	52.48	53.84	37.82	39.06
	0.50	80.16	32.58	32.62	21.16	21.96
UCL		7.04	14.60	14.66	7.04	7.04

Appendix Table 1.8. MCUSUM ATS for uncorrelated (ρ =0.0) Profile Datasets (m=600 , n=20 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	350.16	360.14	357.22	307.02	315.04
	0.10	314.80	328.26	328.54	275.06	280.22
	0.15	273.90	297.94	300.68	241.50	248.94
	0.20	238.24	260.82	268.36	196.56	204.28
	0.25	201.20	229.8	236.52	168.74	176.90
	0.30	172.06	192.36	197.76	140.76	148.36
	0.50	81.96	100.56	105.06	73.66	76.48
0.25	0.00	397.94	320.98	317.28	287.76	290.22
	0.05	348.30	277.12	275.90	255.68	260.00
	0.10	311.12	249.80	250.92	211.58	216.20
	0.15	273.68	203.68	198.70	183.28	191.86
	0.20	236.44	172.74	171.30	151.92	159.66
	0.25	206.60	128.18	122.72	116.20	124.66
	0.30	174.70	101.98	99.68	87.74	88.24
	0.50	79.64	62.64	63.48	53.96	51.16
0.50	0.00	399.28	295.32	285.28	249.42	252.90
	0.05	341.90	251.16	247.88	213.66	211.08
	0.10	300.32	208.86	207.08	188.36	181.50
	0.15	273.08	180.54	178.10	161.96	153.04
	0.20	243.82	150.74	145.90	132.90	126.46
	0.25	216.46	104.08	113.20	100.74	94.20
	0.30	177.44	79.68	80.58	63.98	61.88
	0.50	72.54	46.74	49.06	37.22	35.58
0.75	0.00	393.02	266.96	257.46	227.94	230.92
	0.05	339.18	228.66	225.00	200.18	195.08
	0.10	306.32	199.40	194.06	176.36	171.30
	0.15	273.76	173.54	171.96	148.96	141.46
	0.20	239.82	140.06	141.30	118.20	114.92
	0.25	182.18	108.18	105.38	85.16	86.12
	0.30	140.10	72.46	75.70	50.70	49.84
	0.50	76.46	30.64	33.18	26.72	22.28
1.00	0.00	387.52	234.98	231.18	185.72	188.86
	0.05	343.88	197.86	195.16	161.68	156.92
	0.10	299.64	171.12	166.56	132.30	131.22
	0.15	271.18	141.64	139.36	106.40	105.18
	0.20	238.50	114.52	112.78	78.20	80.96
	0.25	200.76	80.16	74.90	53.96	48.88
	0.30	166.96	50.84	43.86	35.14	30.48
	0.50	85.82	29.12	24.24	21.90	17.40
UCL		7.23	14.40	14.10	7.23	7.23

Appendix Table 1.9. MEWMA ATS for weak auto-correlated (ρ =0.2) Profile Datasets (m=300 , n=10 , l=40)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	346.62	356.40	354.48	327.52	323.38
	0.10	304.70	315.28	309.18	297.52	291.34
	0.15	280.14	290.58	286.14	265.06	258.88
	0.20	255.78	260.44	257.78	240.34	234.16
	0.25	214.16	239.52	236.40	198.72	188.54
	0.30	177.74	195.60	188.66	162.26	156.08
	0.50	104.76	120.44	117.78	88.40	80.38
0.25	0.00	393.28	312.30	307.04	290.34	280.58
	0.05	343.24	267.78	262.72	249.94	237.96
	0.10	301.26	243.42	234.38	229.14	207.34
	0.15	261.38	219.56	219.42	200.94	187.80
	0.20	238.88	182.78	182.76	167.72	158.72
	0.25	195.12	160.52	158.44	142.70	132.74
	0.30	167.78	132.94	131.96	119.22	105.42
	0.50	100.60	85.24	82.72	79.68	68.02
0.50	0.00	397.62	280.44	277.66	256.30	255.18
	0.05	337.42	234.04	230.18	210.78	208.40
	0.10	299.24	207.56	209.86	196.58	197.28
	0.15	255.84	187.4	180.40	158.60	175.96
	0.20	224.50	160.80	152.64	140.78	146.00
	0.25	188.94	140.18	134.36	125.98	120.2
	0.30	159.18	119.36	106.66	100.38	98.76
	0.50	95.92	68.56	65.14	53.46	51.86
0.75	0.00	396.62	249.32	247.10	220.56	219.18
	0.05	335.64	208.22	211.14	178.34	177.56
	0.10	296.48	184.96	187.30	150.36	147.04
	0.15	250.08	169.42	165.40	121.72	119.38
	0.20	216.54	141.58	138.82	109.42	99.98
	0.25	185.84	111.66	111.54	90.46	80.62
	0.30	154.90	87.66	88.62	69.42	60.56
	0.50	101.30	44.94	40.72	34.42	33.44
1.00	0.00	390.70	203.24	202.40	174.70	172.78
	0.05	337.82	181.92	181.90	151.40	146.84
	0.10	287.20	158.64	159.96	131.58	120.76
	0.15	241.18	137.72	140.98	105.54	98.94
	0.20	209.14	121.8	115.40	81.00	71.38
	0.25	185.92	97.74	87.58	62.52	52.00
	0.30	150.94	76.86	61.60	39.98	37.46
	0.50	96.04	37.68	37.54	20.52	19.30
UCL		32.71	46.75	46.60	32.71	32.71

Appendix Table 1.10. MEWMA ATS for weak auto-correlated (ρ =0.2) Profile Datasets (m=300 , n=20 , l=40)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	350.10	357.42	357.54	320.52	319.04
	0.10	321.88	331.34	329.16	289.90	290.50
	0.15	295.54	306.42	302.80	265.50	266.06
	0.20	260.58	272.18	269.76	239.98	240.24
	0.25	223.16	234.98	233.22	200.28	202.42
	0.30	184.24	197.36	193.58	172.22	173.80
	0.50	114.26	128.88	116.02	100.14	96.22
0.25	0.00	395.78	333.42	333.48	291.24	295.04
	0.05	343.62	277.54	273.34	247.14	250.10
	0.10	308.06	241.62	230.70	220.82	215.48
	0.15	287.54	209.20	201.88	198.96	192.60
	0.20	257.62	181.34	173.56	170.74	167.90
	0.25	214.82	159.32	141.20	140.58	132.84
	0.30	181.92	134.74	119.60	118.80	110.82
	0.50	113.80	86.82	76.88	76.02	70.72
0.50	0.00	393.90	299.16	298.56	255.66	260.58
	0.05	331.50	252.80	251.06	213.52	218.96
	0.10	304.02	225.14	222.72	191.62	195.66
	0.15	280.98	200.96	197.24	175.66	168.94
	0.20	250.20	175.28	171.16	159.10	144.74
	0.25	207.80	148.60	140.22	140.06	124.28
	0.30	174.50	126.48	108.62	114.88	107.40
	0.50	113.38	80.86	74.18	67.72	61.04
0.75	0.00	399.00	257.92	255.92	218.96	220.58
	0.05	332.92	207.14	205.62	170.64	171.20
	0.10	300.58	184.22	180.56	144.68	140.48
	0.15	269.26	159.86	154.48	128.42	118.86
	0.20	241.18	133.16	130.26	101.12	93.08
	0.25	197.48	110.66	106.56	77.48	72.58
	0.30	169.00	85.84	80.60	50.58	46.38
	0.50	100.78	51.90	49.16	27.34	22.88
1.00	0.00	397.34	220.56	218.72	168.78	176.80
	0.05	329.40	185.32	180.52	131.66	140.94
	0.10	293.36	169.12	164.70	100.76	104.72
	0.15	261.36	147.52	142.46	86.92	88.92
	0.20	227.58	120.16	118.88	65.18	70.22
	0.25	182.38	90.82	88.46	46.66	48.56
	0.30	150.12	73.02	69.18	30.00	27.60
	0.50	96.56	36.84	35.04	12.62	14.12
UCL		32.85	46.15	46.22	32.85	32.85

Appendix Table 1.11. MEWMA ATS for weak auto-correlated (ρ =0.2) Profile Datasets (m=600 , n=10 , l=70)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	326.98	344.04	338.62	326.98	326.98
	0.10	268.56	299.40	293.66	268.56	268.56
	0.15	237.24	260.72	259.82	237.24	237.24
	0.20	189.38	222.64	218.00	189.38	189.38
	0.25	154.32	196.78	192.12	154.32	154.32
	0.30	121.58	161.34	159.7	121.58	121.58
	0.50	68.10	92.70	92.62	68.10	68.10
0.25	0.00	398.46	320.94	322.08	292.86	285.50
	0.05	330.00	276.24	279.12	260.18	257.90
	0.10	263.08	226.28	222.36	212.70	206.18
	0.15	240.40	196.92	191.92	184.52	181.42
	0.20	196.28	165.62	161.58	151.10	144.42
	0.25	152.60	131.24	131.20	119.68	100.22
	0.30	125.52	101.96	103.82	83.84	72.68
	0.50	71.58	60.40	60.34	40.22	38.26
0.50	0.00	390.34	285.60	284.98	242.60	241.04
	0.05	332.20	255.52	255.52	217.98	212.08
	0.10	269.12	211.92	209.90	191.5	186.86
	0.15	232.6	184.46	183.90	161.24	154.84
	0.20	189.52	153.22	151.76	136.26	129.64
	0.25	156.14	117.06	115.22	107.66	90.92
	0.30	122.40	83.32	81.18	75.58	64.94
	0.50	68.96	49.04	48.98	35.92	32.52
0.75	0.00	388.60	252.94	246.90	218.40	209.88
	0.05	343.08	211.78	210.08	194.08	189.86
	0.10	294.74	186.56	184.56	169.62	163.40
	0.15	259.58	160.34	156.34	137.22	128.86
	0.20	222.46	135.24	131.84	108.74	100.70
	0.25	197.22	103.16	94.10	76.40	68.52
	0.30	142.68	76.82	70.76	50.36	40.78
	0.50	80.96	41.82	41.22	30.06	27.24
1.00	0.00	394.52	235.94	235.90	197.44	194.52
	0.05	347.46	205.74	194.20	165.60	159.02
	0.10	296.88	173.54	167.40	137.46	134.86
	0.15	253.38	147.92	143.74	111.22	109.00
	0.20	221.30	121.32	116.50	76.50	74.84
	0.25	186.08	92.56	90.38	50.22	46.76
	0.30	153.26	64.26	63.98	31.74	28.18
	0.50	79.58	30.70	30.54	14.48	13.18
UCL		31.81	45.00	44.90	31.81	31.81

Appendix Table 1.12. MEWMA ATS for weak auto-correlated (ρ =0.2) Profile Datasets (m=600 , n=20 , l=70)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	348.18	352.86	353.72	342.68	339.50
	0.10	315.02	325.20	325.88	301.34	296.08
	0.15	280.40	290.86	292.78	277.50	279.36
	0.20	252.08	258.64	261.84	238.06	245.28
	0.25	223.26	221.64	229.86	217.86	211.24
	0.30	172.14	171.42	181.06	158.50	155.42
	0.50	78.56	87.66	92.14	72.38	73.78
0.25	0.00	384.14	322.88	330.84	283.74	293.32
	0.05	352.20	283.76	282.48	253.40	257.38
	0.10	290.78	250.60	247.12	222.60	233.22
	0.15	259.26	206.54	205.8	192.58	201.42
	0.20	211.24	172.62	173.3	161.82	159.86
	0.25	180.54	146.96	142.72	133.32	137.70
	0.30	153.62	121.30	110.26	85.98	90.16
	0.50	86.46	63.60	60.30	46.96	48.26
0.50	0.00	390.54	274.2	282.02	240.28	250.98
	0.05	339.10	238.56	249.28	219.74	214.94
	0.10	304.44	200.10	207.38	179.86	171.32
	0.15	249.90	171.42	175.64	151.22	144.78
	0.20	200.06	142.82	136.10	125.14	118.04
	0.25	177.12	112.92	104.90	98.40	91.42
	0.30	144.32	82.06	81.04	71.96	65.24
	0.50	78.34	48.86	52.36	35.24	33.32
0.75	0.00	386.88	229.16	226.06	196.76	190.16
	0.05	340.54	195.42	188.84	169.46	160.36
	0.10	299.46	166.04	165.52	141.02	135.94
	0.15	255.56	140.78	139.68	120.60	111.18
	0.20	210.76	112.70	115.68	98.46	91.00
	0.25	180.08	89.08	85.94	75.88	64.38
	0.30	143.68	60.26	58.64	51.36	48.06
	0.50	77.44	35.92	32.46	24.00	26.82
1.00	0.00	385.26	192.22	192.88	174.44	171.76
	0.05	372.02	160.66	160.30	150.34	147.78
	0.10	330.10	135.08	132.70	132.02	127.34
	0.15	259.82	121.50	117.86	109.54	100.88
	0.20	200.14	97.84	91.86	81.42	74.68
	0.25	172.08	71.68	61.84	53.42	46.50
	0.30	140.56	52.18	43.80	36.90	30.12
	0.50	72.10	26.74	22.72	21.8	19.38
UCL		31.15	45.58	45.50	31.15	31.15

Appendix Table 1.13. MCUSUM ATS for weak auto-correlated (ρ =0.2) Profile Datasets (m=300 , n=10 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	338.04	352.42	349.92	323.98	321.78
	0.10	300.76	309.80	307.72	292.56	286.36
	0.15	276.78	278.90	277.00	258.38	250.18
	0.20	248.56	250.80	246.72	234.22	224.02
	0.25	207.78	225.50	227.88	193.48	191.3
	0.30	168.60	190.38	184.04	161.10	156.88
	0.50	97.48	112.92	109.68	86.42	84.70
0.25	0.00	391.8	298.16	298.04	279.56	277.26
	0.05	339.84	253.88	253.32	235.94	232.62
	0.10	298.64	228.28	227.06	219.78	215.88
	0.15	257.14	196.00	195.54	178.68	174.04
	0.20	230.98	170.70	167.88	154.24	152.54
	0.25	188.94	147.80	133.60	126.20	118.58
	0.30	165.72	121.46	111.16	105.96	97.58
	0.50	95.48	77.74	67.56	69.74	66.78
0.50	0.00	392.38	268.56	266.54	244.80	240.98
	0.05	335.44	224.54	222.24	207.14	205.56
	0.10	289.04	202.70	199.60	190.62	189.36
	0.15	249.02	179.64	175.84	160.84	159.6
	0.20	196.04	156.78	152.64	141.18	137.04
	0.25	164.98	131.88	126.98	119.58	115.78
	0.30	147.48	97.62	97.14	91.26	89.34
	0.50	89.32	61.48	57.42	49.48	47.00
0.75	0.00	397.62	242.18	242.12	217.88	212.92
	0.05	330.62	204.08	200.90	175.60	172.44
	0.10	288.00	178.10	174.10	157.92	154.16
	0.15	245.64	154.06	153.96	140.66	138.36
	0.20	209.56	139.20	137.60	104.68	100.32
	0.25	180.40	111.06	109.18	80.14	76.26
	0.30	150.54	84.84	70.84	61.34	60.44
	0.50	87.32	42.60	38.58	30.46	29.18
1.00	0.00	393.42	192.94	191.94	166.40	163.98
	0.05	333.24	171.20	171.16	148.58	146.92
	0.10	280.40	157.38	152.18	124.54	115.52
	0.15	241.02	130.84	134.96	100.54	91.26
	0.20	207.82	116.44	111.32	78.14	70.62
	0.25	175.38	98.16	95.58	59.90	46.92
	0.30	146.48	70.56	66.54	37.74	30.70
	0.50	84.38	30.82	26.72	19.70	14.70
UCL		7.95	15.08	15.05	7.95	7.95

Appendix Table 1.14. MCUSUM ATS for weak auto-correlated (ρ =0.2) Profile Datasets (m=300 , n=20 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	346.32	353.46	351.26	311.626	321.82
	0.10	315.66	324.06	322.68	280.88	297.48
	0.15	291.58	301.08	299.94	256.94	271.06
	0.20	254.36	267.08	264.72	230.58	236.54
	0.25	214.78	230.34	228.86	202.38	200.94
	0.30	180.32	189.44	186.78	170.32	173.82
	0.50	111.20	113.06	110.92	93.80	94.94
0.25	0.00	396.58	320.78	320.78	286.46	289.14
	0.05	338.78	272.30	271.04	246.40	261.24
	0.10	306.40	249.04	246.64	221.18	230.46
	0.15	284.68	222.06	220.78	191.6	200.96
	0.20	249.04	201.88	197.32	171.20	181.36
	0.25	213.06	175.86	172.60	141.18	159.18
	0.30	176.80	157.02	151.48	115.18	125.38
	0.50	107.04	94.26	90.76	62.48	72.72
0.50	0.00	399.14	292.98	291.04	250.42	250.84
	0.05	328.84	249.42	244.80	204.76	212.92
	0.10	300.78	221.24	219.26	181.96	201.28
	0.15	273.48	192.10	192.98	164.74	189.16
	0.20	249.06	168.38	171.02	149.14	169.58
	0.25	204.92	147.18	144.98	132.80	145.72
	0.30	171.20	126.98	120.42	108.42	121.36
	0.50	108.64	80.76	78.36	55.86	62.50
0.75	0.00	389.14	250.66	250.00	211.78	212.98
	0.05	330.98	200.28	197.18	166.94	188.62
	0.10	298.42	178.28	175.70	142.92	160.74
	0.15	267.14	150.46	144.68	120.22	136.92
	0.20	235.28	126.34	120.94	100.74	104.92
	0.25	188.96	104.24	100.78	71.26	81.06
	0.30	160.24	88.08	78.84	55.10	58.44
	0.50	106.94	51.28	48.02	31.46	37.26
1.00	0.00	393.06	214.80	212.40	181.14	184.42
	0.05	338.76	183.16	179.16	152.82	155.28
	0.10	292.90	164.72	160.54	128.44	131.58
	0.15	267.96	140.78	141.32	107.08	108.66
	0.20	228.12	117.96	118.74	85.78	86.64
	0.25	186.6	90.68	88.82	60.62	62.58
	0.30	152.42	69.46	65.96	38.24	38.62
	0.50	103.90	34.52	33.12	10.02	10.86
UCL		7.82	15.12	15.30	7.82	7.82

Appendix Table 1.15. MCUSUM ATS for weak auto-correlated (ρ =0.2) Profile Datasets (m=600 , n=10 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	332.78	349.48	357.02	332.78	332.78
	0.10	270.04	304.98	310.56	270.04	270.04
	0.15	240.48	269.88	265.26	240.48	240.48
	0.20	202.60	230.18	229.50	202.60	202.60
	0.25	170.70	200.16	198.28	170.70	170.70
	0.30	128.22	167.54	171.92	128.22	128.22
	0.50	75.82	90.26	97.28	75.82	75.82
0.25	0.00	388.20	319.72	320.92	294.86	290.62
	0.05	344.80	281.42	278.20	257.30	251.12
	0.10	284.00	228.70	224.28	215.18	211.00
	0.15	258.88	201.22	194.08	179.1	176.86
	0.20	206.54	168.38	161.06	150.72	150.48
	0.25	162.64	139.52	135.28	120.56	116.02
	0.30	130.02	110.32	109.00	93.74	88.82
	0.50	73.48	61.74	58.90	47.14	45.82
0.50	0.00	399.96	283.56	280.94	252.70	248.80
	0.05	342.78	257.46	252.44	212.78	209.50
	0.10	293.80	215.06	209.38	181.06	175.58
	0.15	246.46	172.16	169.88	153.06	149.34
	0.20	201.42	141.94	138.12	124.48	118.72
	0.25	157.62	114.14	108.98	100.12	91.96
	0.30	131.20	86.46	83.98	73.96	67.86
	0.50	69.20	45.24	40.24	35.52	33.14
0.75	0.00	395.90	259.92	256.64	211.12	207.88
	0.05	339.58	210.32	203.60	180.96	173.70
	0.10	301.54	174.06	175.60	154.96	147.76
	0.15	264.38	148.32	150.96	131.02	123.90
	0.20	215.62	121.02	124.94	105.08	99.80
	0.25	182.60	99.16	97.14	70.70	65.08
	0.30	136.92	68.52	65.68	46.28	40.68
	0.50	71.42	36.28	35.78	28.84	24.86
1.00	0.00	382.92	221.58	214.86	189.84	185.40
	0.05	333.82	193.94	188.20	160.04	155.40
	0.10	286.74	160.42	154.22	132.08	125.94
	0.15	251.88	136.38	131.28	104.12	99.72
	0.20	189.24	111.38	103.82	70.16	65.40
	0.25	157.08	84.82	77.04	45.62	43.06
	0.30	109.94	54.08	49.86	29.44	24.90
	0.50	60.30	32.82	32.06	16.04	14.26
UCL		7.12	14.62	14.35	7.12	7.12

Appendix Table 1.16. MCUSUM ATS for weak auto-correlated (ρ =0.2) Profile Datasets (m=600 , n=20 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	336.48	343.76	340.30	334.12	323.84
	0.10	305.56	315.46	313.82	301.14	290.50
	0.15	269.72	276.6	274.8	262.78	256.80
	0.20	227.48	226.12	224.02	219.64	219.46
	0.25	188.18	193.50	194.18	187.36	183.64
	0.30	156.44	161.30	154.64	154.54	150.02
	0.50	74.84	80.00	79.62	68.04	69.02
0.25	0.00	399.56	322.10	325.50	276.86	272.52
	0.05	355.16	281.62	282.56	240.52	235.66
	0.10	305.56	245.52	249.98	216.36	211.52
	0.15	260.22	204.26	205.70	178.96	172.42
	0.20	225.44	177.52	169.38	146.08	140.14
	0.25	188.32	144.26	143.54	123.18	114.96
	0.30	156.56	117.14	115.72	94.18	85.80
	0.50	74.44	60.08	59.26	51.36	45.94
0.50	0.00	392.04	272.98	279.38	241.46	249.80
	0.05	347.32	234.72	246.72	213.36	215.86
	0.10	298.20	205.94	202.10	187.32	189.92
	0.15	257.02	174.62	173.68	160.90	159.46
	0.20	210.70	144.02	141.60	130.40	128.66
	0.25	176.70	119.64	115.46	106.16	102.34
	0.30	149.38	81.86	82.60	75.70	67.90
	0.50	64.50	41.98	43.58	35.66	33.12
0.75	0.00	396.46	224.48	220.48	191.10	188.38
	0.05	336.10	201.10	194.18	164.96	161.88
	0.10	300.62	177.22	171.94	141.04	135.80
	0.15	258.22	149.88	147.96	109.10	107.94
	0.20	212.48	121.38	118.08	84.74	79.52
	0.25	187.78	91.00	86.52	60.56	55.20
	0.30	151.84	59.62	55.42	34.18	30.84
	0.50	78.90	34.32	33.12	20.50	18.68
1.00	0.00	394.62	196.00	192.44	168.82	165.70
	0.05	359.52	167.98	165.58	140.84	137.78
	0.10	310.90	135.88	133.30	129.12	118.28
	0.15	254.72	110.78	109.52	109.18	100.36
	0.20	201.60	82.00	80.64	81.04	72.06
	0.25	175.68	54.96	53.3	50.76	51.78
	0.30	139.98	38.22	34.98	32.32	27.08
	0.50	72.22	25.54	21.04	17.36	15.80
UCL		7.27	14.36	14.17	7.27	7.27

Appendix Table 1.17. MEWMA ATS for moderate auto-correlated (ρ =0.5) Profile Datasets (m=300 , n=10 , l=40)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	343.70	356.92	359.88	343.70	343.70
	0.10	301.70	311.26	301.56	301.70	301.70
	0.15	265.88	270.26	259.94	265.88	265.88
	0.20	234.46	247.46	226.48	234.46	234.46
	0.25	208.68	216.52	201.78	208.68	208.68
	0.30	181.30	189.12	167.92	181.30	181.30
	0.50	106.54	111.92	100.10	106.54	106.54
0.25	0.00	397.32	330.82	337.70	294.34	296.28
	0.05	339.70	284.62	280.30	264.42	260.84
	0.10	295.96	262.64	261.78	229.98	222.26
	0.15	260.32	224.02	221.46	208.64	200.22
	0.20	225.04	195.78	195.14	191.00	188.40
	0.25	201.28	169.76	167.22	167.42	160.26
	0.30	170.82	139.66	140.22	141.44	142.48
	0.50	102.28	80.80	78.18	71.10	66.46
0.50	0.00	391.48	285.46	284.58	264.84	266.94
	0.05	336.48	222.56	214.64	241.84	236.26
	0.10	285.40	206.88	199.48	207.88	201.88
	0.15	255.80	180.42	173.92	183.86	174.62
	0.20	218.64	163.08	153.84	156.76	151.04
	0.25	190.70	147.34	141.72	130.30	127.86
	0.30	157.30	132.18	122.32	106.98	102.20
	0.50	99.50	75.06	67.82	57.24	52.00
0.75	0.00	396.54	256.46	261.24	220.28	222.52
	0.05	329.52	211.70	216.66	196.36	190.56
	0.10	288.66	191.36	188.44	172.26	164.04
	0.15	246.72	143.34	140.24	139.8	140.48
	0.20	210.98	122.74	117.68	119.16	112.84
	0.25	187.56	99.46	91.12	90.94	89.98
	0.30	153.82	78.14	74.20	64.26	65.04
	0.50	99.32	39.34	33.22	27.18	29.26
1.00	0.00	393.28	211.36	209.24	182.76	180.22
	0.05	324.18	180.24	178.70	165.64	157.26
	0.10	286.92	147.56	151.68	141.94	138.92
	0.15	250.80	118.76	118.16	110.84	110.84
	0.20	209.06	92.46	91.08	87.50	82.48
	0.25	181.00	66.38	64.10	61.60	53.14
	0.30	155.18	40.24	41.52	35.74	29.36
	0.50	100.92	25.14	21.26	18.62	13.08
UCL		33.02	47.00	47.51	33.02	33.02

Appendix Table 1.18. MEWMA ATS for moderate auto-correlated (ρ =0.5) Profile Datasets (m=300 , n=20 , l=40)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	332.82	342.88	341.36	307.00	332.82
	0.10	287.14	280.94	289.58	281.32	287.14
	0.15	261.46	259.06	259.58	250.40	261.46
	0.20	221.02	217.38	219.58	215.96	221.02
	0.25	182.22	188.14	194.14	169.10	182.22
	0.30	151.16	159.08	161.34	141.10	151.16
	0.50	92.76	106.00	100.12	80.70	88.76
0.25	0.00	394.20	334.96	342.4	321.48	326.52
	0.05	330.68	311.98	316.72	289.06	299.86
	0.10	281.16	279.92	279.54	241.40	249.54
	0.15	251.40	244.70	241.02	188.62	200.44
	0.20	208.02	196.36	189.12	163.14	172.22
	0.25	180.20	151.42	136.82	136.72	141.40
	0.30	145.74	114.80	109.94	100.86	105.02
	0.50	88.86	69.56	70.98	54.46	59.62
0.50	0.00	394.18	313.24	319.36	282.56	293.66
	0.05	326.44	274.66	274.18	245.14	250.18
	0.10	285.34	231.66	227.14	213.56	221.46
	0.15	240.92	192.66	190.18	167.62	183.22
	0.20	201.62	140.04	145.76	126.3	144.18
	0.25	172.28	115.22	113.68	100.16	113.74
	0.30	147.58	88.74	84.68	75.50	80.50
	0.50	89.00	51.66	49.20	41.10	43.70
0.75	0.00	397.42	280.40	290.60	255.36	261.90
	0.05	316.50	255.18	263.16	225.42	235.90
	0.10	281.48	200.72	213.54	195.9	204.42
	0.15	244.70	174.94	181.76	167.62	160.40
	0.20	200.80	146.5	138.86	141.20	130.12
	0.25	164.68	105.92	104.14	100.96	99.70
	0.30	138.42	76.52	81.52	64.04	79.34
	0.50	81.08	40.82	43.42	31.30	42.14
1.00	0.00	394.38	272.10	274.18	220.32	228.62
	0.05	323.82	231.78	240.40	188.36	181.08
	0.10	281.74	195.38	192.50	158.58	157.10
	0.15	236.68	170.56	167.46	118.48	128.90
	0.20	201.16	141.98	129.42	86.54	94.82
	0.25	167.02	69.36	81.42	58.52	66.24
	0.30	140.32	40.92	46.72	31.84	35.64
	0.50	83.34	23.04	25.22	13.10	13.46
UCL		32.74	46.31	46.59	32.74	32.74

Appendix Table 1.19. MEWMA ATS for moderate auto-correlated (ρ =0.5) Profile Datasets (m=600 , n=10 , l=70)

γ	δ	T_{P}^{2}	T_{NP}^{2}	T_{NPR}^{2}	T_{SP}^{2}	T_{SPR}^{2}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	332.78	367.40	365.46	332.78	332.78
	0.10	302.44	331.92	329.96	302.44	302.44
	0.15	259.46	292.42	289.88	259.46	259.46
	0.20	216.92	246.18	246.84	216.92	216.92
	0.25	180.58	211.78	204.50	180.58	180.58
	0.30	143.18	176.74	173.70	143.18	143.18
	0.50	79.64	107.42	100.52	79.64	79.64
0.25	0.00	399.88	328.70	324.10	296.70	293.68
	0.05	345.38	291.18	286.92	264.76	258.78
	0.10	308.32	244.48	234.74	231.36	222.44
	0.15	272.34	211.62	207.70	194.22	193.46
	0.20	237.34	173.68	171.40	164.04	160.92
	0.25	192.96	140.44	143.28	130.56	128.58
	0.30	163.84	116.26	113.42	100.96	97.18
	0.50	93.26	69.72	66.20	60.26	59.64
0.50	0.00	392.86	288.84	284.60	260.90	255.74
	0.05	341.30	258.74	252.26	233.96	228.66
	0.10	308.44	217.04	213.54	201.84	194.64
	0.15	272.80	191.40	186.82	174.74	171.58
	0.20	229.60	164.78	160.58	139.62	136.42
	0.25	192.94	136.46	130.08	111.28	112.88
	0.30	161.74	110.68	107.60	86.68	85.14
	0.50	81.64	61.40	57.76	51.88	47.82
0.75	0.00	399.54	250.28	249.32	218.28	210.16
	0.05	348.30	226.22	222.06	192.30	185.88
	0.10	296.16	191.28	188.70	167.84	161.18
	0.15	266.20	163.84	158.46	141.52	134.72
	0.20	217.96	138.96	134.08	116.84	108.34
	0.25	185.04	112.34	110.18	90.60	82.28
	0.30	156.52	86.44	85.18	66.28	58.12
	0.50	73.78	44.22	44.34	34.02	30.78
1.00	0.00	390.08	196.44	196.92	170.52	167.64
	0.05	339.58	174.46	173.70	148.40	147.50
	0.10	288.20	150.32	146.66	118.00	113.92
	0.15	238.28	116.88	112.68	97.42	91.84
	0.20	200.30	91.40	89.08	78.84	69.62
	0.25	173.02	73.86	67.18	56.84	53.56
	0.30	138.04	46.32	43.22	32.84	27.44
	0.50	68.56	28.34	26.64	14.62	12.52
UCL		31.73	44.94	45.28	31.73	31.73

Appendix Table 1.20. MEWMA ATS for moderate auto-correlated (ρ =0.5) Profile Datasets (m=600 , n=20 , l=70)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	350.70	359.78	363.82	330.74	336.78
	0.10	278.24	311.82	314.66	264.96	273.04
	0.15	236.22	268.46	269.96	229.52	233.16
	0.20	198.70	225.22	229.06	185.90	188.70
	0.25	170.60	195.36	193.06	160.02	158.60
	0.30	125.68	150.8	148.06	116.82	113.68
	0.50	66.04	84.70	83.84	60.06	58.04
0.25	0.00	399.14	338.72	336.72	290.40	286.48
	0.05	334.78	300.58	300.22	248.84	239.12
	0.10	301.38	272.22	271.50	212.58	208.46
	0.15	268128	239.88	233.26	199.10	180.58
	0.20	221.16	200.52	196.78	170.78	156.78
	0.25	180.72	166.30	164.32	127.32	120.44
	0.30	150.98	134.50	128.80	88.78	83.38
	0.50	71.46	60.50	62.78	46.56	42.98
0.50	0.00	394.80	308.54	306.78	273.26	265.34
	0.05	316.64	267.52	264.78	229.26	225.26
	0.10	289.24	236.74	235.26	196.74	193.16
	0.15	254.30	199.10	194.62	174.90	167.26
	0.20	210.98	160.86	166.52	144.96	134.22
	0.25	177.14	132.42	136.84	111.56	107.36
	0.30	139.44	97.16	101.30	74.76	70.86
	0.50	69.38	54.74	54.18	41.16	37.06
0.75	0.00	393.54	275.04	272.84	238.96	232.42
	0.05	338.70	240.96	239.04	209.14	198.76
	0.10	291.24	201.16	196.74	180.64	169.34
	0.15	261.16	171.26	167.04	156.84	145.12
	0.20	217.14	140.88	136.94	126.92	117.26
	0.25	189.64	112.84	107.14	100.56	90.46
	0.30	151.68	86.54	80.56	66.42	61.18
	0.50	74.96	48.42	43.56	34.96	33.04
1.00	0.00	398.96	233.46	231.44	200.94	192.94
	0.05	333.46	200.76	198.52	172.84	168.46
	0.10	287.58	169.34	165.26	140.46	136.62
	0.15	255.16	140.54	134.70	114.72	110.64
	0.20	211.24	114.76	111.38	89.24	80.56
	0.25	178.94	80.54	79.72	60.56	54.94
	0.30	149.24	54.96	53.04	38.72	34.76
	0.50	73.46	36.74	31.66	20.74	16.70
UCL		31.16	45.63	45.06	31.16	31.16

Appendix Table 1.21. MCUSUM ATS for moderate auto-correlated (ρ =0.5) Profile (m=300 , n=10 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	339.36	343.74	342.56	339.36	339.36
	0.10	312.58	311.76	308.06	312.58	312.58
	0.15	260.60	276.50	270.54	260.60	260.60
	0.20	226.38	241.28	238.50	226.38	226.38
	0.25	200.42	207.24	202.46	200.42	200.42
	0.30	173.26	183.12	179.60	173.26	173.26
	0.50	100.88	112.72	109.10	100.88	100.88
0.25	0.00	398.40	317.68	320.42	299.92	280.58
	0.05	330.90	280.12	281.40	269.88	241.66
	0.10	305.56	243.48	244.10	241.92	219.44
	0.15	272.86	218.54	212.32	219.52	191.26
	0.20	228.70	191.12	192.02	190.88	176.84
	0.25	198.84	170.86	179.76	161.92	156.22
	0.30	169.28	151.38	154.86	133.14	140.78
	0.50	99.48	93.58	96.50	66.18	71.44
0.50	0.00	397.46	278.82	276.88	251.46	255.02
	0.05	340.76	230.50	231.44	221.18	220.72
	0.10	300.66	204.74	201.88	195.06	194.08
	0.15	272.18	181.52	179.66	168.80	168.72
	0.20	226.24	150.28	155.56	140.38	144.64
	0.25	196.62	123.16	128.54	106.2	111.42
	0.30	173.02	108.86	105.86	81.74	77.10
	0.50	100.24	65.66	63.26	45.68	40.54
0.75	0.00	399.92	237.88	236.58	225.30	228.42
	0.05	337.60	201.06	198.56	176.86	188.24
	0.10	298.86	176.38	172.84	148.52	164.32
	0.15	266.34	138.32	134.18	124.74	134.32
	0.20	230.54	111.42	106.68	100.36	100.36
	0.25	197.92	87.04	85.62	76.40	76.40
	0.30	171.900	64.44	61.72	54.08	56.08
	0.50	96.68	34.42	30.06	29.14	30.30
1.00	0.00	396.02	227.60	220.94	199.68	189.46
	0.05	332.56	200.48	198.66	174.86	164.94
	0.10	275.08	159.58	154.68	150.22	149.14
	0.15	250.08	122.98	121.38	114.06	125.24
	0.20	224.64	95.28	99.26	90.42	88.14
	0.25	192.24	72.58	77.10	68.30	70.86
	0.30	165.28	49.2	51.50	45.88	47.36
	0.50	99.12	30.58	25.74	21.76	22.86
UCL		7.58	14.90	14.75	7.58	7.58

Appendix Table 1.22. MCUSUM ATS for moderate auto-correlated (ρ =0.5) Profile Datasets (m=300 , n=20 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	324.38	338.12	341.80	318.50	324.38
	0.10	281.06	290.34	292.12	278.12	281.06
	0.15	234.90	242.08	240.22	229.02	234.9
	0.20	199.82	203.42	200.84	185.72	199.82
	0.25	167.78	175.52	173.46	159.46	167.78
	0.30	141.30	151.98	139.02	135.92	141.3
	0.50	81.68	86.72	82.48	78.64	81.68
0.25	0.00	399.26	341.82	344.70	301.08	317.48
	0.05	320.06	304.92	301.44	275.88	281.90
	0.10	274.14	270.80	269.44	240.52	259.78
	0.15	230.72	220.40	212.24	201.02	219.88
	0.20	195.64	191.48	187.48	169.84	173.60
	0.25	173.58	159.98	161.62	126.36	143.52
	0.30	149.06	128.60	123.66	98.78	108.64
	0.50	82.10	72.86	70.58	61.64	63.22
0.50	0.00	393.16	303.82	300.52	278.50	276.42
	0.05	311.44	261.64	255.10	241.84	236.80
	0.10	280.42	219.14	214.50	199.70	190.80
	0.15	251.18	183.40	176.08	171.16	166.28
	0.20	212.18	157.22	149.78	139.2	136.02
	0.25	176.80	127.04	118.48	110.9	105.54
	0.30	145.14	91.14	94.12	77.30	69.96
	0.50	79.32	47.28	41.86	35.26	28.54
0.75	0.00	390.92	278.50	275.54	246.54	252.32
	0.05	314.88	248.32	244.76	200.94	200.50
	0.10	276.92	211.12	206.52	168.90	161.22
	0.15	243.70	179.58	177.56	140.92	131.16
	0.20	201.9	151.14	144.10	107.84	106.64
	0.25	176.94	119.90	110.38	79.40	72.38
	0.30	145.90	85.72	81.18	55.90	40.72
	0.50	81.24	41.20	34.82	27.64	22.68
1.00	0.00	392.94	246.36	244.48	213.66	206.78
	0.05	322.66	203.74	207.82	186.00	175.14
	0.10	287.64	159.92	151.22	140.40	131.04
	0.15	240.44	128.08	125.10	117.06	105.36
	0.20	212.14	84.78	89.86	80.70	71.22
	0.25	180.86	61.70	64.80	56.48	54.78
	0.30	151.10	31.92	32.52	26.22	30.42
	0.50	76.88	14.94	14.34	11.04	9.00
UCL		7.81	15.09	14.96	7.81	7.81

Appendix Table 1.23. MCUSUM ATS for moderate auto-correlated (ρ =0.5) Profile Datasets (m=600 , n=10 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	350.24	359.24	356.40	350.24	350.24
	0.10	318.32	328.96	125.92	318.32	318.32
	0.15	272.72	286.44	284.24	272.72	272.72
	0.20	224.02	251.06	255.42	224.02	224.02
	0.25	192.14	213.70	216.74	192.14	192.14
	0.30	144.68	167.86	172.28	144.68	144.68
	0.50	87.60	95.08	96.58	87.60	87.60
0.25	0.00	394.76	317.66	319.44	290.58	294.90
	0.05	340.64	285.78	278.06	256.48	262.84
	0.10	312.34	252.42	248.54	226.32	230.62
	0.15	264.74	215.80	218.18	200.72	198.70
	0.20	235.94	186.70	189.80	174.02	170.02
	0.25	200.50	161.06	159.50	140.14	139.74
	0.30	167.26	132.32	134.84	108.68	113.32
	0.50	81.52	75.74	70.04	61.60	63.14
0.50	0.00	394.04	275.76	270.10	247.76	238.46
	0.05	345.12	247.44	243.58	211.44	206.74
	0.10	310.86	213.16	214.08	176.20	172.28
	0.15	260.00	182.58	185.12	152.08	147.14
	0.20	233.40	156.44	155.98	129.94	118.42
	0.25	196.16	128.84	123.96	101.64	94.72
	0.30	160.68	92.86	87.56	69.36	65.14
	0.50	86.96	59.02	53.14	41.62	34.96
0.75	0.00	391.56	233.28	230.16	198.24	191.16
	0.05	341.38	197.24	197.50	170.22	169.08
	0.10	306.78	171.28	168.88	150.36	142.98
	0.15	262.74	146.92	140.02	126.52	125.22
	0.20	215.66	124.88	115.16	108.36	107.1
	0.25	180.44	100.38	90.04	84.24	80.88
	0.30	146.98	73.52	64.58	50.08	46.70
	0.50	77.50	38.06	32.34	28.62	25.24
1.00	0.00	397.72	194.60	196.62	169.12	171.18
	0.05	337.58	159.30	159.80	151.54	153.36
	0.10	302.18	135.90	136.46	125.64	127.22
	0.15	261.50	117.42	119.78	101.64	95.96
	0.20	216.24	89.00	87.48	75.62	69.48
	0.25	174.92	71.66	67.90	55.42	45.52
	0.30	132.46	52.86	49.22	31.34	23.18
	0.50	70.20	33.72	28.38	17.98	12.28
UCL		7.00	14.49	14.58	7.00	7.00

Appendix Table 1.24. MCUSUM ATS for moderate auto-correlated (ρ =0.5) Profile Datasets (m=600 , n=20 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	336.78	346.54	338.64	308.50	302.40
	0.10	289.04	300.60	288.92	279.06	270.86
	0.15	238.78	258.00	258.78	228.56	221.18
	0.20	201.38	225.18	220.22	194.90	188.48
	0.25	166.50	198.44	194.48	156.82	150.86
	0.30	139.04	155.38	150.78	126.44	120.98
	0.50	70.64	88.58	90.78	60.86	69.96
0.25	0.00	398.90	340.70	338.70	309.26	302.74
	0.05	338.56	296.78	288.50	278.50	270.48
	0.10	306.58	268.56	260.58	227.38	220.86
	0.15	268.84	225.32	223.68	190.78	179.04
	0.20	233.58	189.92	191.04	161.24	149.74
	0.25	181.38	160.78	158.72	134.48	121.12
	0.30	149.84	116.84	115.86	100.22	88.44
	0.50	71.66	63.60	65.48	54.58	45.30
0.50	0.00	391.50	292.64	287.28	261.36	255.04
	0.05	330.68	262.72	259.04	234.84	227.36
	0.10	305.18	233.48	231.26	207.50	194.88
	0.15	267.12	200.96	199.14	176.70	168.74
	0.20	240.76	171.36	167.06	146.94	140.76
	0.25	199.04	137.24	133.48	115.26	109.24
	0.30	142.74	109.36	107.14	88.48	77.74
	0.50	67.24	53.06	49.24	47.24	38.96
0.75	0.00	396.94	262.72	265.12	230.46	228.52
	0.05	335.06	230.48	227.28	204.30	200.78
	0.10	297.18	201.28	198.96	178.64	171.26
	0.15	259.26	174.70	171.26	141.04	133.02
	0.20	211.26	140.96	136.78	117.26	104.72
	0.25	186.92	108.42	104.72	82.92	80.58
	0.30	153.44	79.02	77.28	57.24	52.82
	0.50	69.24	43.28	40.96	34.82	31.24
1.00	0.00	398.96	234.96	232.76	198.70	190.68
	0.05	332.84	204.92	200.74	170.52	160.74
	0.10	290.62	180.36	174.94	140.84	134.62
	0.15	261.16	152.64	147.12	115.36	106.92
	0.20	218.94	120.76	114.76	90.84	81.10
	0.25	191.26	95.12	90.42	56.92	50.84
	0.30	158.74	60.96	54.96	36.92	29.34
	0.50	73.04	33.04	29.04	18.74	12.84
UCL		7.25	14.75	14.44	7.25	7.25

Appendix Table 1.25. MEWMA ATS for strong auto-correlated (ρ =0.8) Profile Datasets (m=300 , n=10 , l=40)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	345.20	360.92	362.94	300.78	295.20
	0.10	310.20	322.98	323.48	265.52	258.20
	0.15	280.18	290.12	292.72	234.44	214.08
	0.20	254.76	261.60	266.00	204.00	188.76
	0.25	210.12	218.16	221.52	181.24	162.12
	0.30	164.10	188.02	187.76	148.22	130.10
	0.50	87.34	111.56	102.60	80.70	73.340
0.25	0.00	392.54	319.78	304.34	275.72	270.02
	0.05	353.40	281.20	272.36	248.66	249.68
	0.10	306.200	247.88	247.80	221.98	213.08
	0.15	279.740	219.66	217.84	189.56	183.22
	0.20	244.76	197.84	198.48	161.90	159.94
	0.25	206.12	162.46	161.20	137.80	134.48
	0.30	168.1	139.76	131.32	112.80	110.24
	0.50	81.8	69.30	66.02	50.78	44.74
0.50	0.00	392.82	269.82	275.72	243.66	251.88
	0.05	354.34	239.72	243.80	203.82	215.98
	0.10	291.80	200.12	205.46	183.60	188.12
	0.15	256.50	179.04	177.04	157.50	159.78
	0.20	219.72	157.00	151.04	130.56	131.48
	0.25	185.84	131.48	125.44	100.06	107.56
	0.30	161.94	101.00	99.00	73.20	71.24
	0.50	89.16	52.92	56.94	37.38	36.38
0.75	0.00	396.34	244.62	240.06	222.80	212.76
	0.05	347.62	210.44	213.98	199.08	186.88
	0.10	280.64	180.52	179.64	166.36	160.16
	0.15	248.00	154.84	155.06	140.00	131.80
	0.20	215.96	126.82	124.32	117.34	103.04
	0.25	181.72	104.78	101.30	99.16	75.16
	0.30	152.46	78.76	75.54	60.36	50.36
	0.50	86.92	43.04	41.80	35.48	27.90
1.00	0.00	390.70	208.58	221.62	199.14	190.74
	0.05	340.88	185.78	192.80	171.66	165.26
	0.10	289.76	170.46	175.28	148.14	139.66
	0.15	258.64	134.38	130.46	125.88	117.16
	0.20	208.78	108.76	105.84	98.90	89.62
	0.25	180.26	85.86	81.72	70.98	60.40
	0.30	146.90	57.74	56.86	42.76	35.78
	0.50	85.80	32.58	34.18	21.94	17.82
UCL		32.85	467.50	47.52	32.85	32.85

Appendix Table 1.26. MEWMA ATS for strong auto-correlated (ρ =0.8) Profile Datasets (m=300 , n=20 , l=40)

γ	δ	T_P^2	T_{NP}^2	T_{NPR}^2	T_{SP}^2	T_{SPR}^2
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	335.94	351.58	347.62	326.40	335.94
	0.10	313.54	319.60	312.96	306.00	313.54
	0.15	265.30	282.72	281.18	260.34	265.30
	0.20	236.50	244.06	250.70	231.70	236.50
	0.25	200.00	214.10	213.38	199.60	200.00
	0.30	171.22	188.48	178.18	168.14	171.22
	0.50	103.44	126.14	118.90	99.62	103.44
0.25	0.00	392.32	366.88	366.40	329.98	320.64
	0.05	350.10	317.28	313.84	282.60	279.12
	0.10	322.80	290.76	280.42	255.74	251.5
	0.15	281.82	258.12	251.42	221.54	217.26
	0.20	254.46	230.04	220.58	191.48	187.24
	0.25	205.62	183.48	172.18	158.40	157.58
	0.30	169.02	146.56	139.14	124.84	126.84
	0.50	101.68	79.40	76.54	59.28	51.66
0.50	0.00	398.14	325.90	315.84	284.48	281.48
	0.05	339.04	297.36	290.44	260.88	254.70
	0.10	309.70	264.04	260.34	232.64	221.36
	0.15	280.28	239.32	234.34	201.54	193.70
	0.20	246.12	203.28	200.60	174.28	167.70
	0.25	206.56	166.40	158.82	137.32	129.34
	0.30	173.00	139.92	129.88	110.06	100.96
	0.50	98.28	60.96	57.96	49.20	43.12
0.75	0.00	399.16	302.96	303.98	259.46	255.68
	0.05	348.02	261.54	258.10	221.48	220.76
	0.10	299.70	232.40	226.26	199.04	191.48
	0.15	268.64	202.70	193.44	171.28	162.46
	0.20	241.78	179.28	175.50	125.02	118.64
	0.25	204.08	146.28	138.48	92.34	85.00
	0.30	168.14	101.26	96.10	67.54	58.12
	0.50	103.70	53.54	50.90	35.06	29.26
1.00	0.00	397.42	257.88	252.66	217.16	204.98
	0.05	340.12	218.92	215.70	183.04	171.48
	0.10	296.44	191.56	188.90	153.40	147.06
	0.15	257.14	161.68	161.20	129.04	119.30
	0.20	218.94	134.96	131.52	100.32	94.32
	0.25	174.56	111.6	107.44	73.82	65.36
	0.30	146.98	77.10	74.22	45.30	39.72
	0.50	84.54	41.26	38.70	23.50	21.36
UCL		32.83	46.49	47.52	32.83	32.83

Appendix Table 1.27. MEWMA ATS for strong auto-correlated (ρ =0.8) Profile Datasets (m=600 , n=10 , l=70)

γ	δ	T_{P}^{2}	T_{NP}^{2}	T_{NPR}^{2}	T_{SP}^{2}	T_{SPR}^{2}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	340.34	355.16	356.04	313.72	330.52
	0.10	304.98	321.90	317.82	280.48	294.22
	0.15	276.66	292.52	290.38	250.14	260.72
	0.20	232.96	250.72	257.04	216.50	228.58
	0.25	198.72	221.34	224.56	193.96	198.24
	0.30	162.9	185.42	182.12	147.48	154.50
	0.50	86.48	103.96	106.74	77.04	84.86
0.25	0.00	398.76	329.44	328.36	286.20	284.56
	0.05	368.32	302.12	301.42	260.48	255.12
	0.10	334.02	248.52	249.70	234.54	227.30
	0.15	287.04	207.46	200.66	201.12	201.68
	0.20	240.96	180.18	176.18	170.52	174.98
	0.25	211.84	152.26	145.62	143.38	134.36
	0.30	176.94	123.62	118.62	114.34	112.90
	0.50	96.46	73.34	71.14	58.54	61.36
0.50	0.00	399.82	274.98	269.28	241.90	232.50
	0.05	364.20	236.74	241.42	215.18	203.66
	0.10	330.90	201.44	212.72	190.56	177.52
	0.15	265.24	175.62	165.32	164.62	141.80
	0.20	239.62	138.22	141.96	130.12	117.18
	0.25	185.62	111.54	108.94	99.20	90.96
	0.30	155.10	81.40	81.04	72.34	63.90
	0.50	88.72	47.72	49.12	40.14	34.32
0.75	0.00	389.94	230.28	228.88	190.50	188.78
	0.05	357.90	199.64	192.74	169.96	163.98
	0.10	303.78	171.84	159.80	155.90	148.10
	0.15	266.12	139.08	131.48	129.30	117.46
	0.20	228.94	109.64	105.64	99.12	97.00
	0.25	200.22	76.50	75.86	68.64	66.58
	0.30	149.16	60.00	55.98	52.50	44.20
	0.50	80.52	34.14	32.54	31.00	27.88
1.00	0.00	387.08	196.40	196.92	174.44	175.76
	0.05	338.30	174.76	175.00	158.34	153.22
	0.10	280.20	156.00	151.64	140.50	135.08
	0.15	259.46	133.06	131.52	117.74	115.10
	0.20	204.32	108.14	112.56	93.40	90.74
	0.25	181.80	81.18	82.84	64.50	64.52
	0.30	147.10	57.42	58.02	46.06	46.28
	0.50	74.68	29.58	28.68	24.30	27.14
UCL		31.90	45.34	45.24	31.90	31.90

Appendix Table 1.28. MEWMA ATS for strong auto-correlated (ρ =0.8) Profile Datasets (m=600 , n=20 , l=70)

γ	δ	T^2_P	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	354.00	362.74	361.24	354.00	338.50
	0.10	322.40	335.22	332.42	322.40	310.52
	0.15	291.28	299.22	297.56	291.28	275.56
	0.20	262.54	268.28	270.60	262.54	244.22
	0.25	224.44	238.28	236.16	224.44	201.90
	0.30	170.96	178.28	176.04	170.96	166.52
	0.50	77.64	86.06	84.90	77.64	69.18
0.25	0.00	398.80	321.48	319.04	298.82	294.02
	0.05	351.68	292.36	290.14	268.50	263.68
	0.10	317.18	258.96	258.34	240.28	233.48
	0.15	289.28	226.70	222.30	209.60	201.48
	0.20	249.08	184.06	193.06	176.34	172.50
	0.25	215.92	152.96	152.36	142.34	141.86
	0.30	173.74	121.52	116.56	113.42	106.78
	0.50	81.76	65.66	62.56	58.94	49.66
0.50	0.00	393.64	286.88	282.44	260.54	251.10
	0.05	336.48	260.24	257.80	228.26	222.94
	0.10	301.98	233.58	228.66	200.14	190.72
	0.15	270.58	195.66	191.70	179.46	170.22
	0.20	220.20	165.22	160.36	152.92	145.92
	0.25	188.18	138.86	135.76	119.12	109.9
	0.30	154.18	108.70	107.68	88.76	85.00
	0.50	77.90	55.88	51.74	46.52	40.00
0.75	0.00	388.78	238.58	238.02	197.66	190.00
	0.05	341.60	196.88	196.24	173.40	167.60
	0.10	294.10	169.12	167.72	149.20	143.26
	0.15	264.00	134.56	132.10	126.70	114.76
	0.20	230.76	103.28	103.36	100.56	90.52
	0.25	192.36	76.76	76.34	68.62	66.34
	0.30	156.92	53.68	51.96	41.80	40.20
	0.50	78.00	31.94	29.98	24.88	22.90
1.00	0.00	391.94	186.68	186.34	155.06	147.58
	0.05	353.30	155.68	154.92	141.14	128.00
	0.10	291.94	133.78	132.98	115.10	107.82
	0.15	250.20	109.78	108.24	92.54	89.64
	0.20	219.04	85.28	84.66	68.16	64.94
	0.25	186.30	69.70	66.28	57.82	48.88
	0.30	152.62	46.96	45.26	35.96	27.18
	0.50	71.28	26.14	23.98	18.46	15.72
UCL		31.66	45.10	45.03	31.66	31.66

Appendix Table 1.29. MCUSUM ATS for strong auto-correlated (ρ =0.8) Profile Datasets (m=300 , n=10 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	342.60	353.00	349.58	296.60	283.14
	0.10	304.92	307.90	305.90	254.92	245.04
	0.15	272.00	271.60	272.36	212.00	203.16
	0.20	241.42	231.08	234.80	181.42	174.84
	0.25	217.44	202.08	205.92	157.44	148.98
	0.30	159.84	174.02	181.36	129.84	118.42
	0.50	85.16	99.94	100.80	64.20	60.74
0.25	0.00	393.20	293.54	293.20	264.84	261.38
	0.05	338.74	259.68	261.34	238.02	231.48
	0.10	306.94	230.94	237.90	213.98	206.60
	0.15	273.70	202.02	201.42	189.56	181.26
	0.20	247.44	181.84	179.92	160.92	158.58
	0.25	211.40	159.98	156.70	139.50	131.00
	0.30	163.82	134.18	131.88	110.60	102.42
	0.50	91.06	62.52	58.50	54.82	51.04
0.50	0.00	398.56	259.98	251.94	235.82	225.38
	0.05	334.06	231.52	221.26	208.44	200.70
	0.10	297.68	186.70	182.26	176.42	169.48
	0.15	249.82	163.50	165.46	150.90	143.68
	0.20	210.46	141.52	140.50	123.76	119.86
	0.25	180.34	117.60	116.22	99.30	93.58
	0.30	152.30	93.34	83.26	75.00	67.40
	0.50	83.58	52.68	47.28	40.34	31.06
0.75	0.00	392.98	233.32	231.04	213.38	212.90
	0.05	344.00	205.32	199.24	197.66	193.62
	0.10	294.46	174.60	170.30	167.54	159.34
	0.15	243.12	153.64	145.28	137.56	135.50
	0.20	210.22	116.22	113.40	115.42	109.54
	0.25	181.26	90.16	86.16	89.22	85.22
	0.30	155.62	66.76	62.74	63.04	57.04
	0.50	89.22	38.92	36.10	30.96	24.96
1.00	0.00	394.30	206.46	205.64	167.56	161.62
	0.05	333.48	180.26	176.22	149.64	141.42
	0.10	274.20	155.48	153.48	133.08	125.40
	0.15	251.02	120.02	127.76	108.96	101.20
	0.20	201.50	94.94	88.88	84.68	79.10
	0.25	179.14	68.38	62.30	56.50	52.88
	0.30	150.40	44.32	40.32	34.20	30.38
	0.50	89.82	28.74	26.72	18.84	15.38
UCL		7.74	14.82	14.80	7.74	7.74

Appendix Table 1.30. MCUSUM ATS for strong auto-correlated (ρ =0.8) Profile Datasets (m=300 , n=20 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	327.52	340.44	343.42	318.50	327.52
	0.10	297.28	298.92	293.96	292.36	297.28
	0.15	251.90	262.18	257.96	238.08	251.90
	0.20	209.02	220.58	219.12	201.24	209.02
	0.25	172.80	192.04	191.08	167.76	172.80
	0.30	144.70	150.52	149.06	138.98	144.70
	0.50	89.00	94.66	94.04	86.38	89.00
0.25	0.00	399.98	366.76	364.74	321.08	316.92
	0.05	341.30	321.62	311.88	277.84	271.68
	0.10	308.70	299.24	285.78	241.98	235.66
	0.15	282.22	251.66	250.02	209.44	201.86
	0.20	255.94	217.80	220.04	182.22	173.48
	0.25	209.14	189.28	187.02	155.32	148.64
	0.30	173.72	143.34	141.82	127.02	120.40
	0.50	98.64	75.16	72.46	50.64	44.70
0.50	0.00	395.34	319.30	314.16	277.56	271.46
	0.05	341.64	291.90	283.68	247.94	241.28
	0.10	310.76	262.18	254.36	215.58	207.38
	0.15	283.10	231.30	224.06	189.52	175.70
	0.20	255.02	194.48	191.76	160.82	151.56
	0.25	219.44	161.28	156.58	131.32	107.34
	0.30	179.70	124.76	123.16	94.68	70.58
	0.50	100.04	64.90	61.16	42.98	39.56
0.75	0.00	399.28	284.24	284.40	248.02	252.64
	0.05	345.34	250.84	244.84	214.16	221.06
	0.10	306.36	219.50	218.38	188.30	192.20
	0.15	276.76	193.90	191.44	157.94	160.10
	0.20	233.62	160.46	162.58	131.70	124.70
	0.25	200.44	128.54	127.32	95.44	100.46
	0.30	165.92	101.36	94.84	63.22	60.98
	0.50	97.88	48.04	44.26	37.46	30.60
1.00	0.00	397.62	255.84	256.00	211.98	205.16
	0.05	337.88	224.40	227.26	181.96	171.78
	0.10	300.86	200.98	199.94	152.20	142.84
	0.15	261.24	165.44	164.00	120.16	115.46
	0.20	219.04	136.84	132.32	88.28	81.80
	0.25	194.08	111.98	109.72	56.00	54.18
	0.30	167.02	69.48	70.10	31.66	29.28
	0.50	100.68	41.32	39.80	17.98	17.04
UCL		8.06	14.67	14.73	8.06	8.06

Appendix Table 1.31. MCUSUM ATS for strong auto-correlated (ρ =0.8) Profile Datasets (m=600 , n=10 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	340.74	351.16	352.62	308.74	319.34
	0.10	288.02	313.80	320.06	274.02	281.04
	0.15	246.12	260.86	285.78	228.12	239.20
	0.20	200.46	220.98	240.22	190.46	198.46
	0.25	174.84	188.96	202.44	158.84	170.84
	0.30	139.38	159.56	163.80	121.38	129.38
	0.50	81.40	88.44	84.76	69.40	72.30
0.25	0.00	397.54	327.76	300.30	275.56	271.82
	0.05	356.74	294.70	258.12	247.42	241.56
	0.10	328.06	257.34	224.36	215.84	213.84
	0.15	281.24	208.12	199.44	190.46	186.92
	0.20	232.46	181.24	171.88	167.76	160.92
	0.25	201.42	142.00	138.04	134.56	129.92
	0.30	159.40	109.02	105.88	109.86	103.30
	0.50	93.48	68.98	64.34	61.32	58.38
0.50	0.00	393.94	273.70	270.28	232.28	228.06
	0.05	344.18	239.60	237.28	206.12	199.90
	0.10	318.12	198.38	209.12	178.76	175.24
	0.15	277.8	173.36	182.88	152.78	149.04
	0.20	240.36	144.38	154.06	130.30	127.80
	0.25	211.50	119.50	125.44	102.34	100.46
	0.30	164.20	92.98	85.02	87.86	76.10
	0.50	93.06	56.20	50.90	49.72	43.38
0.75	0.00	395.26	224.34	238.94	190.56	187.10
	0.05	335.90	194.44	190.82	166.52	167.04
	0.10	300.06	163.02	159.06	138.94	141.56
	0.15	264.14	136.74	132.16	112.92	117.22
	0.20	220.22	105.06	101.70	94.50	94.72
	0.25	180.38	72.34	69.94	70.68	69.04
	0.30	144.22	56.72	53.70	50.30	46.74
	0.50	84.84	33.02	36.52	28.68	25.14
1.00	0.00	390.08	192.64	184.30	165.70	161.66
	0.05	350.00	164.30	160.66	141.86	141.72
	0.10	299.10	143.30	141.38	120.24	127.46
	0.15	267.54	114.76	117.16	104.18	102.26
	0.20	241.56	90.86	87.54	83.90	85.68
	0.25	199.18	67.24	68.26	59.92	59.86
	0.30	135.64	50.56	53.84	37.70	41.78
	0.50	77.52	28.34	32.62	20.26	22.80
UCL		7.12	14.77	14.74	7.12	7.12

Appendix Table 1.32. MCUSUM ATS for strong auto-correlated (ρ =0.8) Profile Datasets (m=600 , n=20 , l=70)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	400.00	400.00	400.00	400.00	400.00
	0.05	353.00	364.50	361.18	353.00	337.00
	0.10	309.62	329.70	327.86	309.62	299.62
	0.15	270.10	287.06	280.52	270.10	262.10
	0.20	244.86	253.48	255.10	244.86	230.86
	0.25	203.32	213.96	201.64	203.32	193.32
	0.30	164.3	180.84	175.38	164.30	156.30
	0.50	74.68	91.06	87.26	74.68	64.68
0.25	0.00	393.08	325.66	321.42	275.98	272.22
	0.05	337.48	293.40	291.38	251.76	247.72
	0.10	293.40	260.72	257.54	216.04	212.18
	0.15	250.22	238.32	233.46	179.88	171.92
	0.20	211.70	197.90	199.22	153.28	151.18
	0.25	181.92	163.82	159.76	124.88	120.62
	0.30	161.16	127.48	124.54	93.92	89.70
	0.50	76.56	66.88	65.72	52.06	47.24
0.50	0.00	396.94	274.54	272.50	247.02	241.94
	0.05	338.78	243.92	236.94	220.80	215.86
	0.10	288.28	208.70	206.84	193.30	190.24
	0.15	257.72	173.12	168.86	163.18	156.10
	0.20	212.12	148.9	142.58	135.04	129.64
	0.25	181.20	114.10	111.30	102.26	96.72
	0.30	150.88	80.66	74.76	71.72	68.32
	0.50	80.40	51.62	48.68	39.66	35.42
0.75	0.00	398.68	243.52	240.40	210.20	206.92
	0.05	337.72	215.96	210.84	190.40	183.08
	0.10	302.62	172.92	171.72	164.76	157.84
	0.15	264.88	149.10	152.92	130.66	127.42
	0.20	221.22	122.44	122.12	104.58	99.02
	0.25	178.62	89.52	87.50	70.16	66.54
	0.30	140.78	63.88	60.18	45.76	40.10
	0.50	73.92	36.68	37.86	28.34	24.16
1.00	0.00	385.20	203.46	200.42	168.48	165.62
	0.05	331.78	180.38	179.00	148.70	146.32
	0.10	298.50	154.42	152.30	120.26	113.40
	0.15	261.50	139.52	135.78	92.28	85.36
	0.20	221.80	112.96	111.90	68.06	64.88
	0.25	187.44	81.38	80.00	47.94	44.82
	0.30	139.78	53.24	52.32	29.78	24.60
	0.50	71.72	33.74	31.18	15.14	13.44
UCL		7.15	14.63	14.81	7.15	7.15

Appendix Table 1.33. MEWMA SDRL for un-correlated (ρ =0.0) Profile Datasets for (m=300 , n=10 , l=40)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	99.94	101.26	102.47	96.25	95.26
	0.05	91.32	93.31	93.44	89.09	88.81
	0.10	83.32	86.77	85.19	82.93	80.92
	0.15	74.79	78.41	76.28	72.82	73.89
	0.20	65.79	74.71	69.09	63.57	60.7
	0.25	55.17	62.34	61.23	55.35	52.01
	0.30	48.19	50.21	53.68	47.43	45.68
	0.50	30.20	37.65	35.17	31.47	29.65
0.25	0.00	101.95	95.20	94.53	90.05	87.41
	0.05	92.70	87.28	85.28	81.07	76.87
	0.10	85.61	82.92	80.12	68.86	69.24
	0.15	79.55	75.41	74.53	60.49	57.14
	0.20	64.50	62.45	61.30	49.55	50.14
	0.25	56.34	55.34	54.64	38.07	39.35
	0.30	49.28	47.06	46.84	27.79	28.97
	0.50	34.90	34.62	32.63	25.10	23.74
0.50	0.00	101.28	80.24	82.45	75.15	74.18
	0.05	92.67	73.57	74.28	70.12	67.03
	0.10	84.13	65.54	64.06	62.14	57.81
	0.15	76.09	60.89	59.58	57.17	54.97
	0.20	63.92	51.43	52.21	50.94	47.93
	0.25	56.99	45.78	44.69	43.22	40.84
	0.30	47.46	40.79	39.28	38.89	36.13
	0.50	34.82	26.59	25.22	24.54	23.43
0.75	0.00	100.96	71.37	72.12	67.23	64.87
	0.05	92.29	64.58	63.53	57.09	55.65
	0.10	84.42	56.95	55.25	50.08	49.79
	0.15	76.16	51.38	51.71	43.17	42.82
	0.20	64.73	45.96	43.01	36.38	35.76
	0.25	53.10	37.62	36.49	32.15	28.67
	0.30	46.09	31.99	30.05	25.89	22.11
	0.50	32.57	20.83	20.70	14.61	13.37
1.00	0.00	99.45	65.85	64.92	57.25	54.92
	0.05	91.95	57.05	55.09	50.21	47.59
	0.15	71.08	42.09	40.25	37.21	35.75
	0.20	63.66	31.89	32.58	32.47	29.73
	0.25	56.42	27.58	25.70	24.17	22.61
	0.30	49.38	23.94	22.87	19.98	16.11
	0.50	34.07	17.63	16.60	12.62	10.42
UCL		32.77	46.53	46.60	32.77	32.77

Appendix Table 1.34. MCUSUM SDRL for weak auto-correlated (ρ =0.2) Profile (m=300 , n=20 , l=40)

γ	δ	S_P	S_{NP}	S_{NPR}	S_{SP}	S_{SPR}
0.00	0.00	98.01	111.56	112.71	95.25	98.25
	0.05	87.01	94.71	96.10	86.97	85.01
	0.10	76.04	86.25	86.57	71.88	72.57
	0.15	60.07	71.22	70.51	58.29	59.42
	0.20	53.85	64.28	62.80	46.38	45.67
	0.25	45.57	56.65	57.21	33.05	32.57
	0.30	37.08	45.74	43.54	26.56	27.42
	0.50	26.16	32.81	34.36	20.30	23.13
0.25	0.00	98.02	90.92	90.81	83.58	87.03
	0.05	86.81	81.32	82.75	76.59	79.92
	0.10	75.11	72.14	72.50	68.48	70.95
	0.15	60.83	60.12	58.18	57.57	58.63
	0.20	51.64	49.85	46.84	45.47	46.17
	0.25	43.45	39.28	36.73	37.11	35.83
	0.30	35.09	30.60	29.90	26.80	25.52
	0.50	23.17	21.07	20.01	16.79	14.29
0.50	0.00	99.78	74.29	70.50	61.87	65.13
	0.05	88.07	65.45	63.30	55.23	57.50
	0.10	76.97	57.90	56.11	49.37	50.04
	0.15	63.68	46.94	44.87	40.65	42.99
	0.20	54.39	39.39	38.30	32.23	34.78
	0.25	37.52	30.26	29.92	23.93	24.71
	0.30	30.94	25.51	25.37	18.74	17.55
	0.50	24.23	19.12	17.16	14.35	13.27
0.75	0.00	105.17	61.66	61.60	52.45	54.39
	0.05	93.44	54.88	53.67	45.59	46.99
	0.10	85.17	47.62	45.14	37.55	40.21
	0.15	69.31	40.14	38.55	29.33	34.42
	0.20	61.25	32.23	31.30	24.80	28.26
	0.25	52.53	25.82	24.53	16.76	20.18
	0.30	40.60	18.36	19.13	10.52	14.05
	0.50	27.94	13.30	14.04	7.33	8.96
1.00	0.00	100.13	57.64	56.46	45.22	47.21
	0.05	89.47	51.38	50.74	40.22	42.74
	0.10	76.55	37.65	36.05	34.29	36.74
	0.15	69.97	32.21	29.95	28.36	30.75
	0.20	55.04	25.76	24.66	23.02	21.4
	0.25	42.19	22.27	20.98	15.95	13.4
	0.30	35.43	15.34	16.67	9.63	8.24
	0.50	26.69	11.85	10.50	6.66	5.32
UCL		7.82	15.12	15.30	7.82	7.82

Appendix Table 1.35. MEWMA SDRL for strong auto-correlated (ρ =0.8) Profile Datasets (m=600 , n=20 , l=70)

γ	δ	T^2_{P}	T^2_{NP}	T^2_{NPR}	T^2_{SP}	T^2_{SPR}
0.00	0.00	154.34	160.70	162.70	154.34	150.31
	0.05	146.62	151.82	152.36	146.62	142.47
	0.10	127.68	133.68	135.36	127.68	125.42
	0.15	119.37	120.45	123.08	119.37	114.68
	0.20	105.10	107.25	108.40	105.10	102.16
	0.25	93.18	98.07	100.81	93.18	90.29
	0.30	84.08	88.10	87.75	84.08	79.24
	0.50	51.45	60.47	61.50	51.45	48.29
0.25	0.00	155.85	139.00	140.23	127.84	125.45
	0.05	143.91	125.43	126.74	113.16	112.36
	0.10	128.24	110.32	112.27	107.64	105.34
	0.20	103.47	74.13	74.85	70.01	74.83
	0.25	93.26	66.97	63.81	61.22	60.69
	0.30	79.09	59.65	57.12	56.17	54.23
	0.50	49.46	40.50	39.58	34.93	33.10
0.50	0.00	158.02	121.35	121.98	115.87	113.88
	0.05	145.54	109.84	110.64	106.33	103.82
	0.10	130.64	94.68	94.39	96.32	94.48
	0.15	123.22	86.67	84.36	84.79	80.44
	0.20	110.24	70.46	69.82	67.21	64.13
	0.25	100.52	62.55	60.02	52.44	47.93
	0.30	87.39	56.18	54.35	45.57	39.67
	0.50	52.78	33.08	34.49	28.91	24.68
0.75	0.00	163.97	112.71	113.55	100.08	97.58
	0.05	149.17	102.74	100.47	94.94	87.50
	0.10	135.88	85.63	88.25	80.59	75.49
	0.15	122.86	65.20	61.89	57.36	53.61
	0.20	106.06	51.46	48.46	43.17	40.55
	0.25	94.02	30.00	30.47	27.14	23.44
	0.30	83.81	21.67	19.29	19.06	15.35
	0.50	43.49	15.98	14.05	13.39	12.62
1.00	0.00	161.60	107.51	106.92	93.82	92.70
	0.05	148.07	96.49	97.39	84.75	83.18
	0.10	131.08	82.37	85.63	73.96	70.15
	0.15	116.42	64.44	65.42	55.92	47.14
	0.20	100.51	48.18	45.61	40.81	36.13
	0.25	87.85	24.64	25.08	23.78	24.02
	0.30	74.01	13.46	14.35	15.72	12.90
	0.50	43.15	10.37	10.49	9.26	7.25
UCL		31.73	44.94	45.28	31.73	31.73